Vladislav Bevc

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UNITED STATES
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Research Paper No. 47

February 1965

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RESEARCH PAPER NO. 47

This work was supported in part by the Office of Naval Research

ACKNOWLEDGMENT

The research leave and the financial support of the Office of Naval Research as well as the use of the U. S. Naval Postgraduate School Computer Facility is gratefully acknowledged.

BEHAVIOR OF SEPARATION CONSTANTS
FOR FINITE GYROMAGNETIC PLASMAS

Abstract

The type of the functions appearing in the expression for electromagnetic fields in finite magnetoplasmas depends only on the properties of the medium manifested in the separation constants. The $\omega - \beta$ plane of the Brillouin diagram is partitioned into a definite number of zones characterized by the type of field solutions. The general quantitative behavior of the separation constant for finite magnetoplasma in a longitudinal magnetic field on the Brillouin diagram is described.

BEHAVIOR OF GYROTROPIC PLASMA SEPARATION CONSTANTS AS FUNCTIONS IN THE ω-β PLANE

Of the numerous papers[1] concerned with the propagation of waves through waveguides filled with gyrotropic plasma columns the majority is confined either to the quasi-static approximation[2,3,4,5] or to a discussion of the problem in general terms with tabulation of auxiliary functions[7], such as, e.g., the biradial functions[8] and an outline of the general method of approach for obtaining the solution[6,7,9,10] without a systematic and comprehensive presentation and classification of the numerous types of ω - β propagation diagrams and field solutions such as are known, e.g., for empty waveguides or waveguides filled with the dielectrics[11] and for slow wave structures.[12] Only a few representative solutions have been published thus far[13,14,15] and they did not include a complete systematic survey of all possible cases although they were certainly oriented in this direction.

Despite a high number of combinations that can arise with the several parameters that can be varied, viz., the plasma density or the corresponding plasma frequency ω_p, the intensity of the collimating magnetic field or the corresponding cyclotron frequency ω_{cy}, and the diameter of the waveguide or the corresponding empty waveguide cutoff frequency, it is nevertheless possible to establish a system of classification of modes of propagation.

It is believed that one way to make it possible readily to obtain the actual solutions for given conditions in waveguides is to prepare

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catalogues where quantitites of interest such as, e.g., the arguments of Bessel functions, biradial functions, and the like are tabulated and presented graphically on charts and diagrams as functions in the $\omega - \beta$ plane for various combinations of values of the plasma frequency, cyclotron frequency, and the cutoff frequency of the empty waveguide. In addition to these, actual solutions for combinations of the above values in reasonable and suitably chosen steps would also be catalogued.

The objective of this paper is to present a study of the separation constants, if indeed these quantities may be called by this name, which in circular cylindrical geometry appear as arguments of Bessel functions in the expressions for the electromagnetic fields[6,7,9,10], viz.,

$$E_z = \frac{c_1 J_n(u_1 \frac{r}{a}) + c_2 J_n(u_2 \frac{r}{a})}{u_1^2 - u_2^2} \tag{1}$$

$$H_z = \frac{c_1 y(u_1) J_n(u_1 \frac{r}{a}) + c_2 y(u_2) J_n(u_2 \frac{r}{a})}{u_1^2 - u_2^2} \tag{2}$$

Evidently, the separation constants determine the kind of the Bessel functions in Eqs. 1 and 2. If either u_1 or u_2 or both are imaginary, the corresponding Bessel function become modified, I_n instead of J_n. If they are complex conjugates Beseel functions of complex arguments appear in the above expressions. It is therefore believed that it is more

useful to know the behavior of the separation constants u_1 and u_2 in the $\omega - \beta$ plane than merely the behavior of the separate components of the equivalent dielectric tensor $\underline{\underline{\epsilon}}$ which is sometimes presented.

The constants u_1 and u_2 are solutions of a biquadratic equation which is obtained solely from the properties of the medium. In addition, of course, they must like, c_1 and c_2, also be compatible with the boundary conditions. The medium or electronic equation was first derived by Hahn[6] and we shall give it in its original form:

$$\left[\left(\frac{u}{a}\right)^2 + \beta^2 + k_p^2 - k^2\right]^2 = \frac{k_{cy}^2}{\left(k^2 - k_p^2\right)}\left[\left(\frac{u}{a}\right)^2 + \beta^2 - k^2\right]\left[\left(\frac{u}{a}\right)^2 + \left(1 - \frac{k_p^2}{k^2}\right)\left(\beta^2 - k^2\right)\right] \quad (3)$$

The following notation is being used here:

$$k_p = \frac{\omega_p}{c} \quad (4\text{-a})$$

$$\omega_p = \sqrt{\frac{e\rho_0}{m\epsilon_0}} \quad (4\text{-b})$$

$$k_{cy} = \frac{\omega_{cy}}{c} \quad (4\text{-c})$$

$$\omega_{cy} = \mu_0 \frac{e}{m} H_0 \quad (4\text{-d})$$

$$k = \frac{\omega}{c} \quad (4\text{-e})$$

Symbols denoting the quantities are as follows:

ω = frequency at which the waves in waveguide propagate;

ρ_0 = electronic volume charge density;

β = propagation constant appearing in the expression $\exp j(\omega t-\beta z)$;

c = velocity of light;

e/m = 1.76×10^{11} coulomb/Kg, electron charge to mass ratio;

H_0 = axial static magnetic field strength;

$\mu_0 = 1.25 \times 10^{-6}$ henry/m;

E_0 = 8.86×10^{-12} farad/m;

a = radius of the cylindrical waveguide.

The explicit solution of the biquadratic equation is:

$$\left(\frac{u_{1,2}}{a}\right)^2 = \frac{1}{2(k^2-k_p^2-k_{cy}^2)}\left\{\left[2(k^2-k_p^2-k_{cy}^2)+\frac{k_p^2k_c^2}{k^2}\right]\left(\beta^2-k^2\right)-2\left(k^2-k_p^2\right)k_p^2\right.$$

$$\left.\pm\frac{k_{cy}^2k_p^2}{k^2}\sqrt{(\beta^2-k^2)^2+4\beta^2\frac{k^2}{k_{cy}^2}(k^2-k_p^2)}\right\} \qquad (5)$$

The plus sign in front of the square root will be consistently used for the calculation of u_1^2 and the negative sign for the calculation of u_2^2.

It is evident that we may multiply the Equation (5) by a^2 throughout and then obtain u^2 expressed in terms of ka, k_pa, $k_{cy}a$, βa which is useful for representation on diagrams since these values are then pure

numbers.

The following relationship holds between the function u^2 and the function χ^2 used by Suhl and Walker[10]:

$$\chi^2_{1,2} = \frac{u^2_{1,2}}{(ka)^2 - (k_p a)^2} \tag{6}$$

It appears that the reason for the use of this expression by Suhl and Walker was the desire for unified treatment of both ferrite and plasma filled waveguides. Apparently propagation was not expected for values of frequency lower than the value of plasma frequency. A numerical solution of the equations of Suhl and Walker would have discovered the plasmaguide modes that were later found by Trivelpiece and Gould[2].

Camus and Le Mezec[14] use the notation:

$$T^2 = \frac{u^2}{a^2} \tag{7}$$

and

$$Z = \frac{u^2}{(k_p a)^2} \tag{8}$$

When boundary conditions are introduced it is best to normalize all frequencies to the frequency defined by:

$$\omega_0 = c/a \tag{9}$$

This means that quantitites $k_p a$, k_{cy}, ka can be considered essentially as ω_p, ω_{cy}, ω normalized with respect to ω_0. Whenever boundary conditions are introduced the radius of the waveguide appears in the expression and both plasma frequency ω_p and cyclotron frequency ω_{cy} are specified by $k_p a$ and $k_{cy} a$ respectively, i.e., in terms of the normalizing frequency $\omega_0 = c/a$. The solution of the boundary condition problem depends on the absolute value of ω_p and ω_{cy}. However, when we consider only the quantities u_1^2 and u_2^2 as functions of the frequency ω and wave number β, i.e., as functions in the $\omega-\beta$ plane it is possible to normalize all frequencies to the plasma frequency (or alternatively to the cyclotron frequency) and the ratio of the cyclotron frequency to the plasma frequency becomes the only parameter that is left to be varied. This ratio is denoted in the following way:

$$M = \frac{\omega_{cy}}{\omega_p} = \frac{(k_{cy} a)}{(k_p a)} \tag{10}$$

Likewise, it is convenient to normalize the other quantitites with respect to the plasma frequency:

$$Y = \frac{(ka)}{(k_p a)} = \frac{\omega}{\omega_p} \tag{11}$$

$$X = \frac{\beta a}{(k_p a)} = \frac{\beta c}{\omega_p} \tag{12}$$

In this normalization the Hahn's equation for u_1^2 and u_2^2 reads:

$$U_{1,2}^2 = \frac{u_{1,2}^2}{(k_p a)^2} = \frac{1}{2(Y^2-1-M^2)}\left\{-\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right](X^2-Y^2)-2(Y^2-1)\right.$$

$$\left.\pm\frac{M^2}{Y^2}\sqrt{(X^2-Y^2)^2+\frac{4X^2Y^2}{M^2}(Y^2-1)}\right\} \qquad (13)$$

Note that U_1^2 and U_2^2 are multivalued at the origin $(X=0, Y=0)$. Its value depends on the path in the Y-X plane along which we approach the origin. It is seen without difficulty that the values of $U_{1,2}^2$ are real for values of X and Y exterior to the region in which the expression under the square root becomes negative, viz.,

$$0 < Y < 1 \qquad (14)$$

$$\frac{Y}{M}\left[\sqrt{1+M^2-Y^2} - \sqrt{1-Y^2}\right] < X < \frac{Y}{M}\left[\sqrt{1+M^2-Y^2} + \sqrt{1-Y^2}\right] \qquad (15)$$

Within this region the values of U_1^2 and U_2^2 are complex conjugates. The boundary of the complex region is a closed curve touching the line $Y = 1$ at $X = 1$ and the line $X = \sqrt{\frac{1+M^2}{M^2}}$ at $Y = \sqrt{\frac{1+M^2}{2+M^2}}$. As M is increased the complex region becomes narrower. This boundary is also shown on Figures 1 to 4 and 6 to 15. On the above boundary curve

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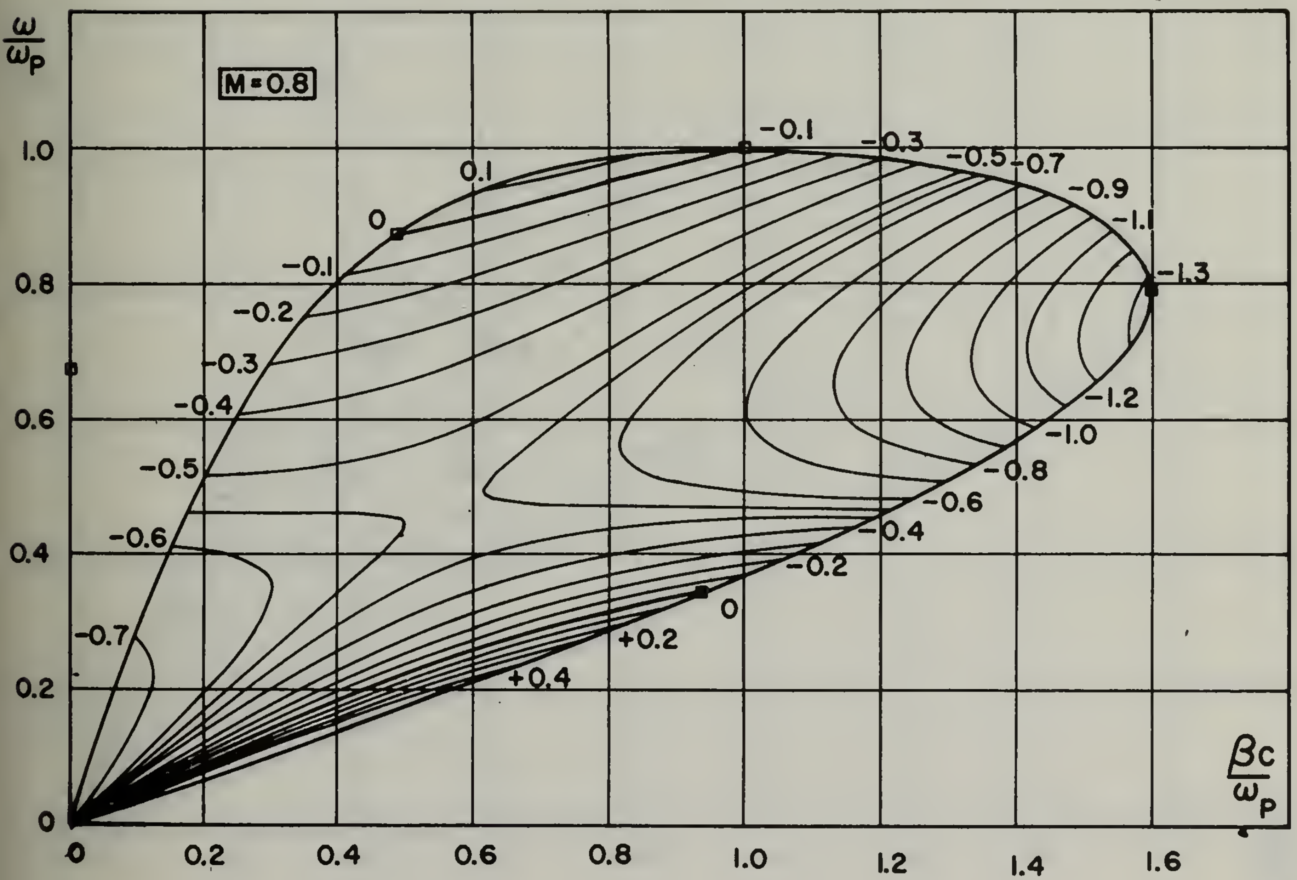
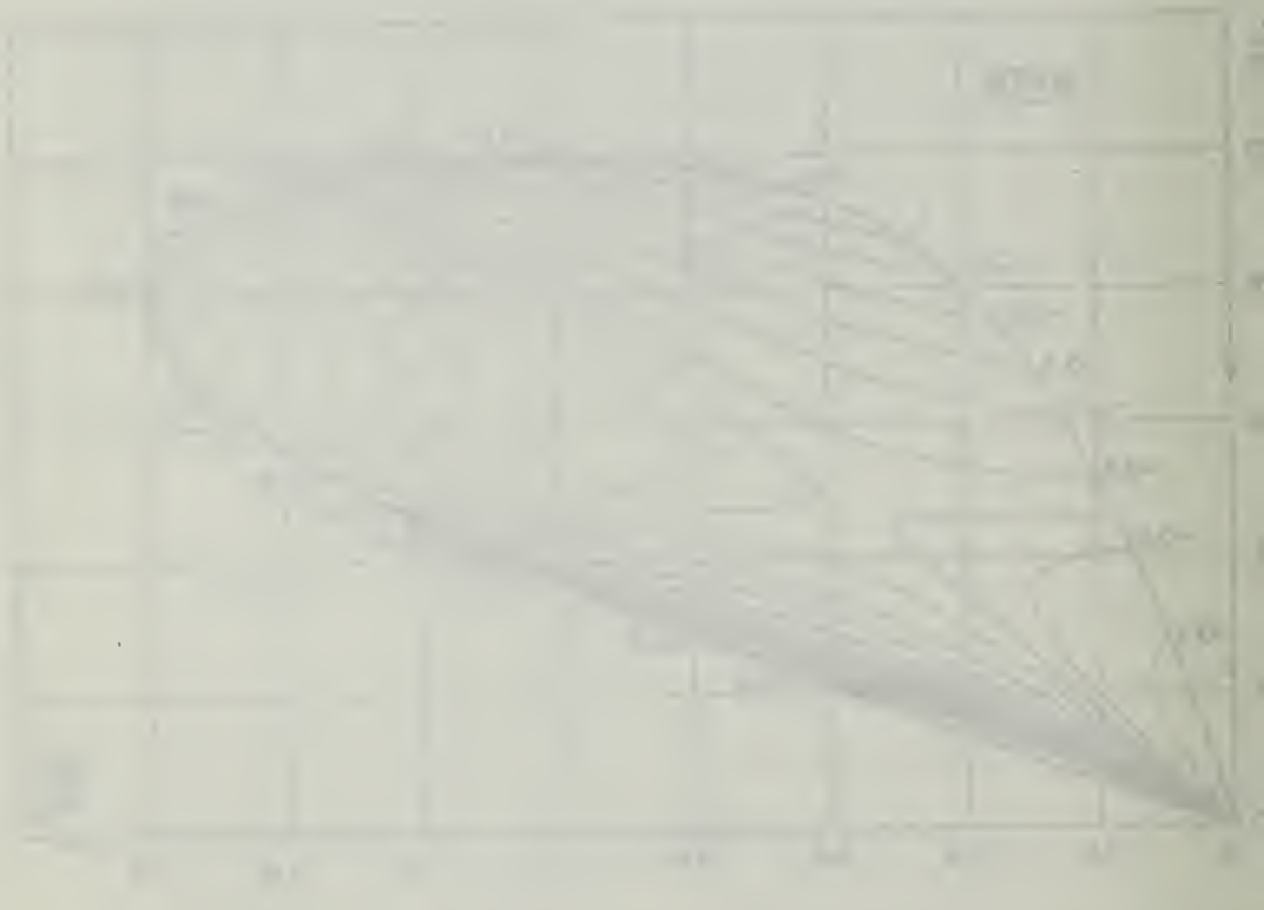


Fig. 1 Contour curves for constant values of $Re(U^2)$ in the complex region of the $\omega - \beta$ plane at the value of $M = 0.8$.



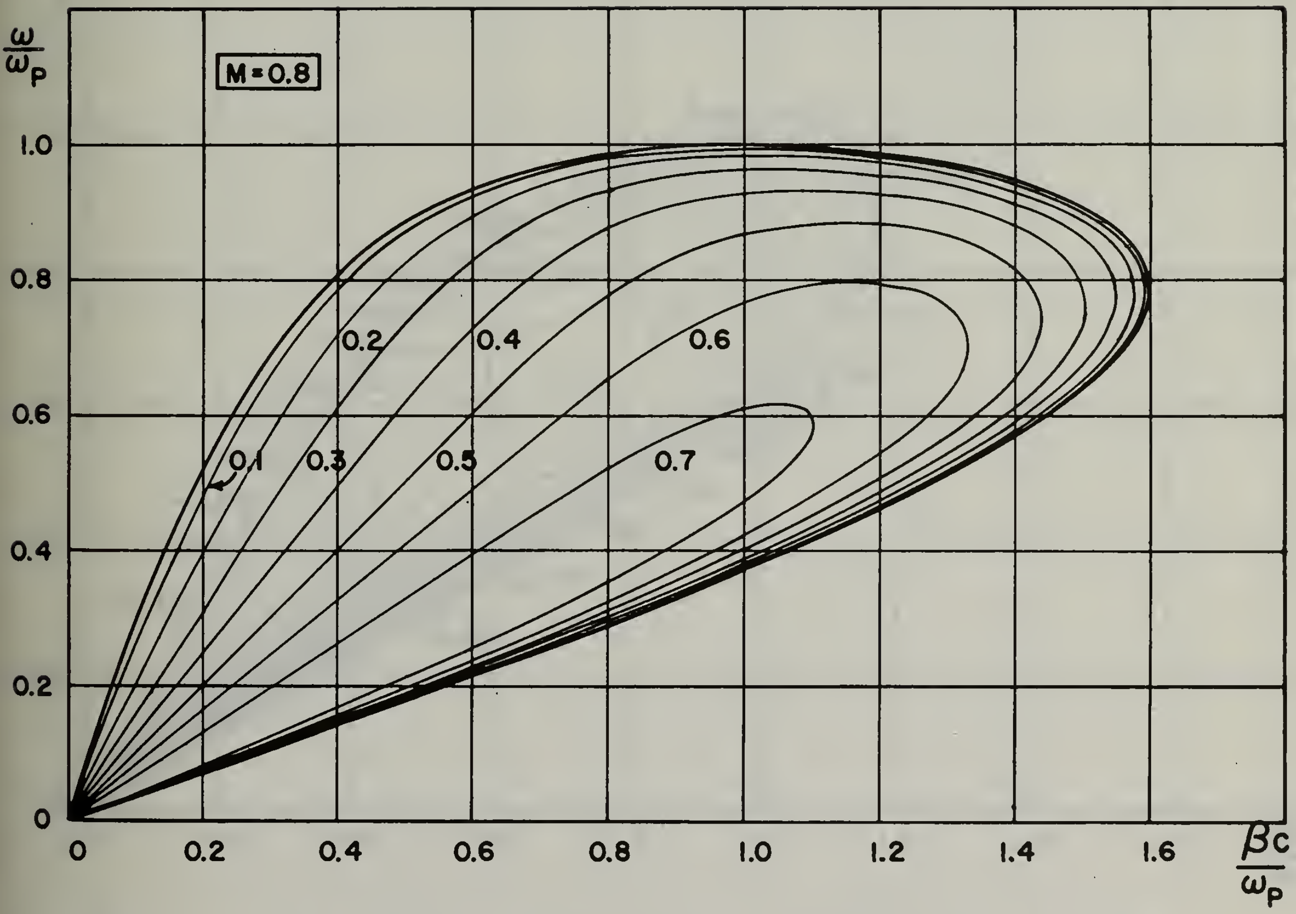


Fig. 2 Contour curves for constant values of $\mathrm{Im}(U^2)$ in the complex region of the $\omega - \beta$ plane at the value of $M = 0.8$.

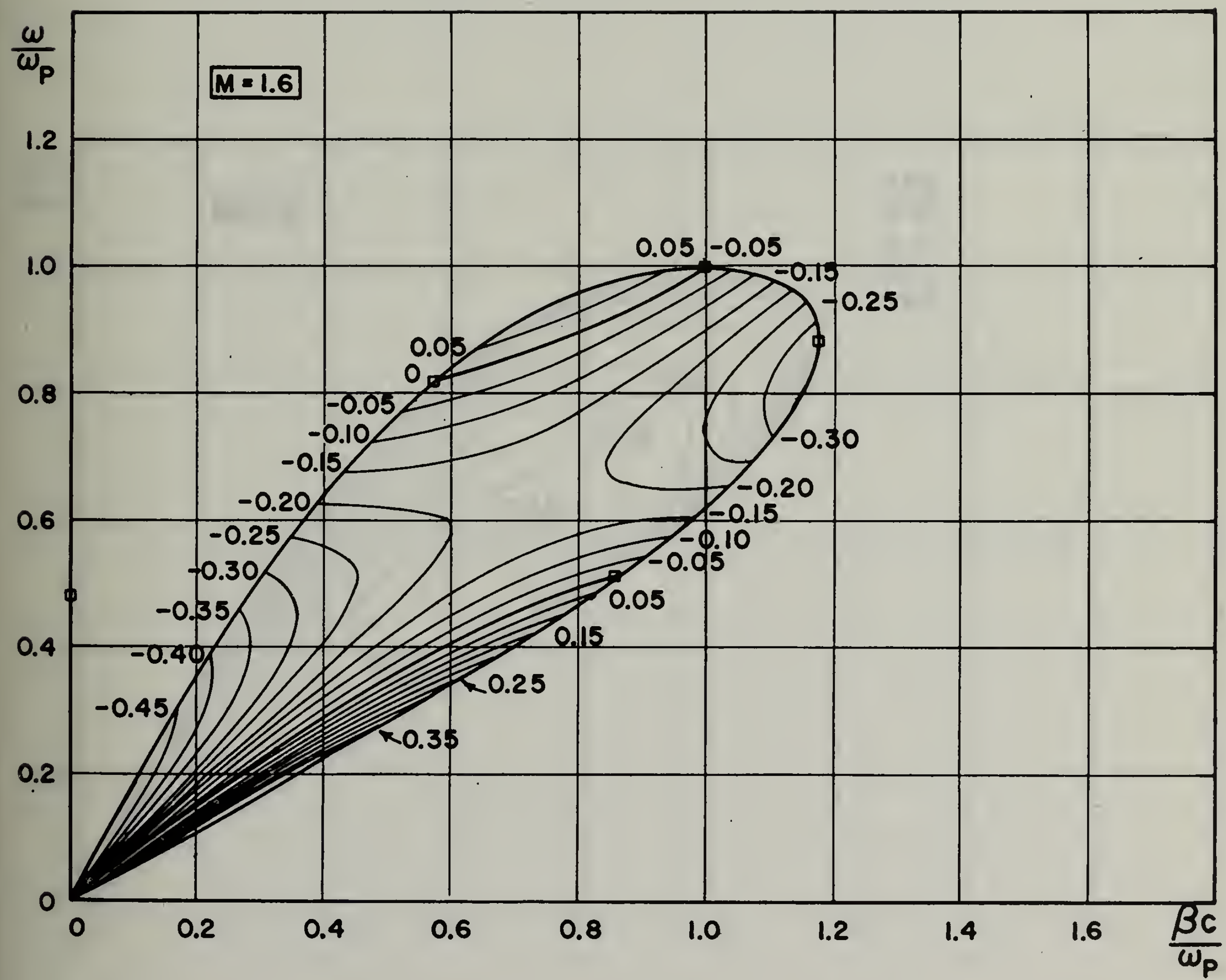


Fig. 3 Contour curves for constant values of $Re(U^2)$ in the complex region of the ω - β plane at the value of $M = 1.6$.

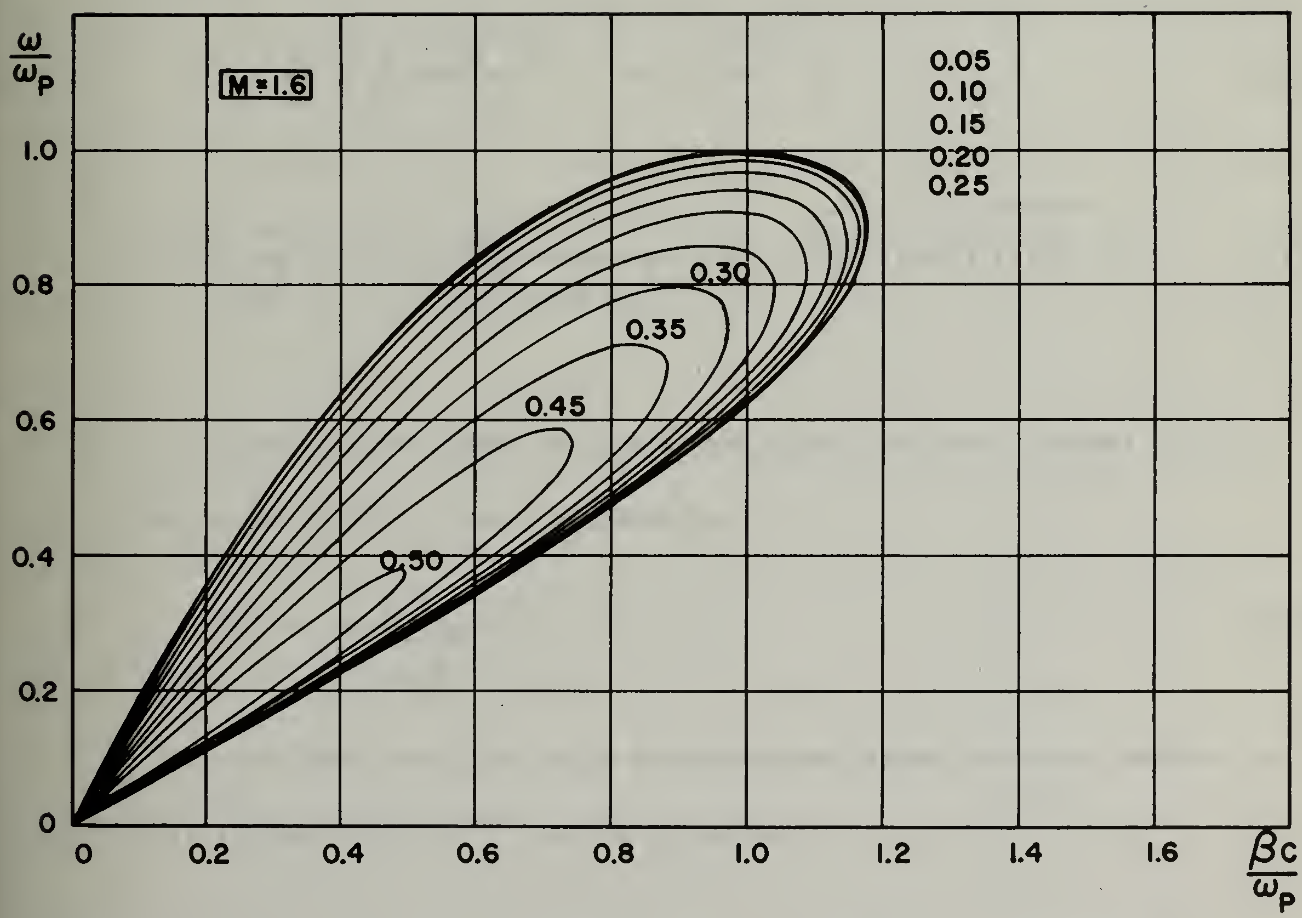
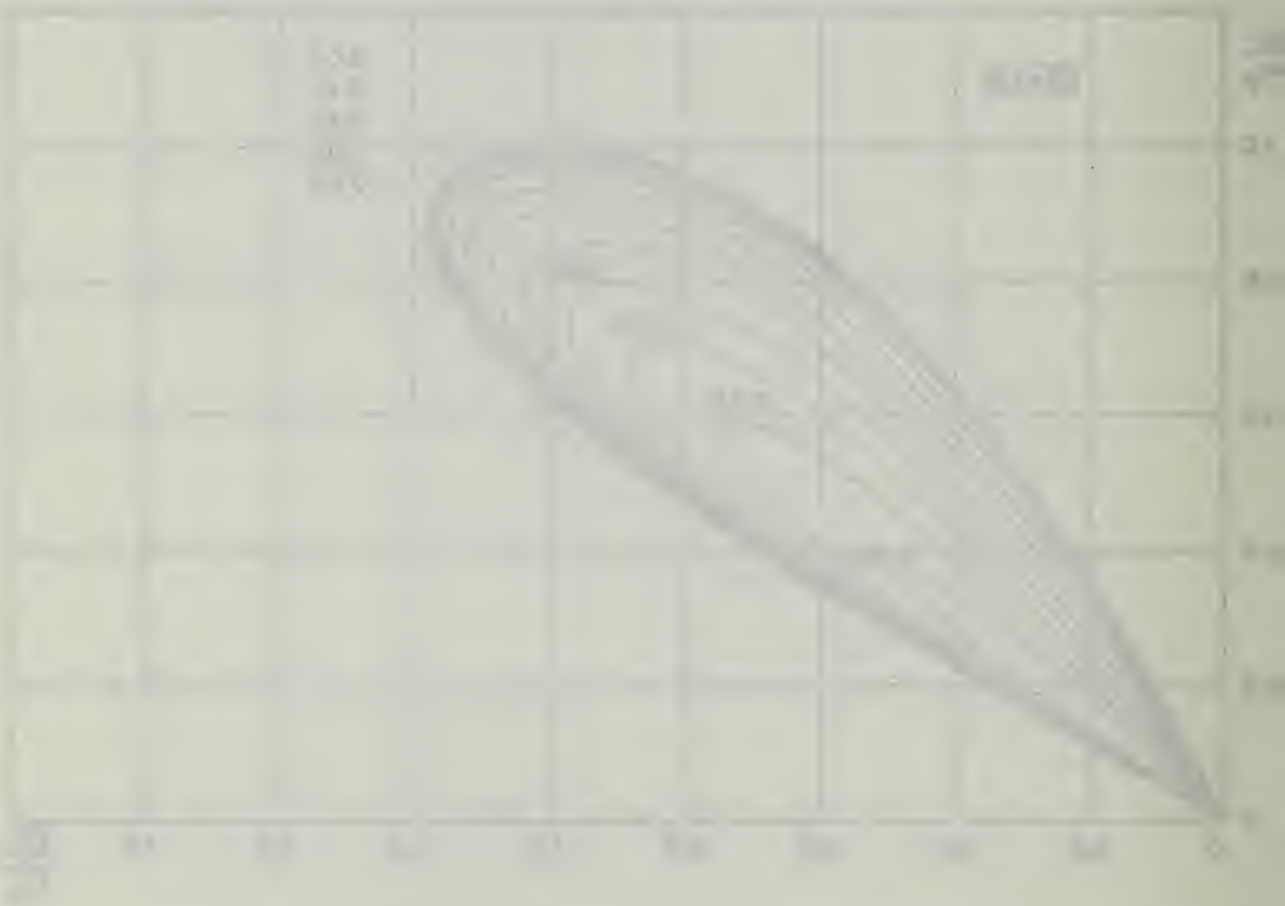


Fig. 4 Contour curves for constant values of $\mathrm{Im}(U^2)$ in the complex region of the $\omega - \beta$ plane at the value of $M = 1.6$.



(Eqs. 14 and 15) the expression under the square root vanishes and $U_1^2 = U_2^2$. The values of $U_1^2 = U_2^2$ on the boundary curve are:

on $$X = \frac{Y}{M}\left[\sqrt{1 + M^2 - Y^2} - \sqrt{1 - Y^2}\right] \tag{16}$$

$$U^2 = 2\frac{Y^2}{M^2}\left\{Y^2 - 1 - \left[\frac{M^2}{2Y^2(1+M^2-Y^2)} - 1\right]\sqrt{(M^2+1-Y^2)(1-Y^2)}\right\} \tag{17}$$

and on

$$X = \frac{Y}{M}\left[\sqrt{1+M^2-Y^2} + \sqrt{1-Y^2}\right] \tag{18}$$

$$U^2 = 2\frac{Y^2}{M^2}\left\{Y^2 - 1 + \left[\frac{M^2}{2Y^2(1+M^2-Y^2)} - 1\right]\sqrt{(M^2+1-Y^2)(1-Y^2)}\right\} \tag{19}$$

If the origin (X=0, Y=0) is approached along the curve defined in Eq. 16 the value of U^2 at zero is given by

$$U^2 = -\frac{1}{\sqrt{1+M^2}} \tag{20}$$

if on the other hand the origin is approached along the curve defined in Eq. 18 the value of U^2 at zero is given by

$$U^2 = +\frac{1}{\sqrt{1+M^2}} \tag{21}$$

The equations of contour lines $Re(U^2)$ = const and $Im(U^2)$ = const within the region defined by Eq. 14 and 15 are readily found. For $Re(U^2)$ = constant

$$X = \sqrt{Y^2 + 2\,\frac{(1-Y^2)+(1+M^2-Y^2)Re(U^2)}{\left[\frac{M^2}{Y^2} - 2(M^2+1-Y^2)\right]}} \qquad (22)$$

for $Im(U^2)$ = constant

$$X = \frac{Y}{M}\sqrt{(M^2+1-Y^2)+(1-Y^2)\pm 2\sqrt{(1-Y^2)(1+M^2-Y^2)-(M^2+1-Y^2)^2(Im(U^2))^2}} \qquad (23)$$

The curves given by Eq. 22 are single valued functions of Y while the curves given by Eq. 23 are double valued; they are closed curves similar to the boundary of the region. Figures 1 to 4 show representative contour diagrams inside the complex region. It is further of interest to determine whether and in which points on the boundary curve the value of U^2 vanishes. Evidently $U^2=0$ at $Y=1$ and $X=1$, other points are found by setting the expressions of Eq. 17 or 19 equal to zero, and arrange the terms in powers of Y^2. Thus we obtain:

$$Y^6-(2+M^2)Y^4+(1+M^2)Y^2 - \frac{M}{4} = 0 \qquad (24)$$

The roots of this equation are real as would become apparent if Eq. 24 were solved; it is possible to obtain them by solving them numerically. However,

a more elegant way to study its roots is available. It will be recalled that either U_1^2 or U_2^2 can vanish on the following two curves:

$$X = \sqrt{Y^2 - \frac{1}{1-\frac{M}{Y}}} \tag{25}$$

$$X = \sqrt{Y^2 - \frac{1}{1+\frac{M}{Y}}} \tag{26}$$

These two curves lie outside of the complex region. Equation 25 represents a curve with a discontinuity at $Y = M$, its upper branch intercepts the Y axis (corresponding to ω axis) at $Y= + \frac{M}{2} + \sqrt{\frac{M^2}{4} + 1}$;

Eq. 26 represents a continuous curve with Y intercept at $Y = - \frac{M}{2} + \sqrt{\frac{M^2}{4} + 1}$. Values of U_1^2 and U_2^2 on curves given by Equations 25 and 26 can be readily evaluated. On the curve given by Eq. 25 we have:

$$U^2_{1,2} = \frac{M}{Y(Y^2-1-M^2)}\left[\frac{Y^3-MY^2-Y+M/2}{Y-M} \pm \sqrt{\left(\frac{Y^3-MY^2-Y+M/2}{Y-M}\right)^2}\right] \tag{27}$$

Care must be exercised in taking the square root in this expression. We shall here take the square root as the opposite operation of squaring. The expression under the square root is always positive but the expression squared can be either positive or negative. For the sign of the square root we take the sign of the squared quantity which we must first evaluate.

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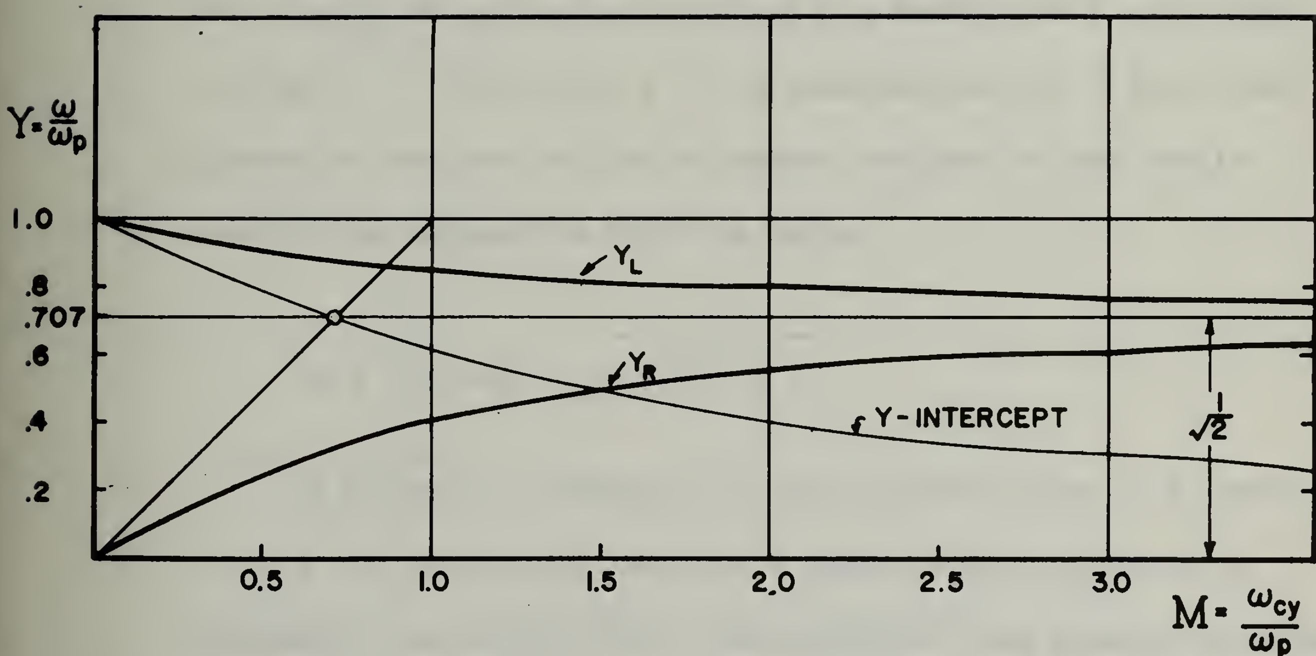

Fig. 5 Plot of the parameter M as a function of $Y(=\omega/\omega p)$ illustrating the solutions of the cubic of Eq. 28.

The roots of the cubic equation

$$Y^3 - MY^2 - Y + \frac{M}{2} = 0 \tag{28}$$

may be determined as follows. From Eq. 28 we have

$$M = \frac{Y(Y-1)(Y+1)}{(Y - \frac{1}{\sqrt{2}})(Y + \frac{1}{\sqrt{2}})} \tag{29}$$

The value of M can be readily plotted as a function of Y as is shown on Fig. 5 . From Fig. 5 it is seen that one root of Eq. 28 designated as first root, is always negative and that the third root is always found between the following limits:

$$M < Y_3 < \frac{M}{2} + \sqrt{(\frac{M}{2})^2 + 1} \tag{30}$$

If we limit our interest to the first quadrant of the $\omega - \beta$ plane ($Y > 0$, $X > 0$) and to real values of X these two roots need not be considered. The second root, on the other hand, lies between the values

$$0 < Y_2 < \frac{1}{\sqrt{2}} \tag{31}$$

This means that a point exists on the lower branch of the curve given by Eq. 25 with an ordinate Y which satisfies the cubic of Eq. 28 and as a consequence both U_1^2 and U_2^2 are equal to zero in that point. If both

U_1^2 and U_2^2 are equal to zero then they are also equal to each other and consequently lie on the curve given by Eq. 18, i.e., on the right boundary of the region of complex values of U_1^2 and U_2^2. Hence the curve of Eq. 25 has one point in common with the right boundary of the complex region given by Eq. 18 and the two curves are tangent to each other in that point. We shall call this point the lower tangent point. Now for values of Y between zero and the second root Y_2 of Eq. 28 the squared expression in Eq. 27 is negative, viz.,

$$Y^3 - MY^2 - Y + \frac{M}{2} > 0 \tag{32}$$

$$Y - M < 0 \tag{33}$$

if $$0 < Y < Y_2 \tag{34}$$

From where follows for U_1^2

$$U_1^2 = \frac{M}{Y(Y^2-1-M^2)} \left\{ \frac{Y^3 - MY - Y + M/2}{Y-M} + \left(-\frac{Y^3 - MY - Y + M/2}{Y-M} \right) \right\} = 0 \tag{35}$$

and for U_2^2:

$$U_2^2 = \frac{2M(Y^3 - MY - Y + M/2)}{Y(Y^2-1-M^2)(Y-M)} \tag{36}$$

When, on the other hand, the value of Y is greater than the value of the second root the following relations hold:

$$Y^3 - MY^2 - Y + \frac{M}{2} < 0 \tag{37}$$

$$Y - M < 0 \tag{38}$$

$$Y_2 < Y < M \tag{39}$$

The squared quantity in Eq. 27 is then positive and we have

$$U_1^2 = \frac{2M(Y^3 - MY^2 - Y + M/2)}{Y(Y^2 - 1 - M^2)(Y - M)} \tag{40}$$

$$U_2^2 = \frac{M}{Y(Y^2 - 1 - M^2)} \left\{ \frac{Y^3 - MY^2 - Y + M/2}{Y - M} - \frac{Y^3 - MY^2 - Y + M/2}{Y - M} \right\} = 0 \tag{41}$$

Thus it is seen that on the curve given by Eq. 35 between the origin ($X=0$, $Y=0$) and the point in which this curve touches the boundary of the complex region the function U_1^2 is zero and the function U_2^2 has a value given by Eq. 40, on the rest of the curve, beyond the lower tangent point, the situation is reversed and the function U_2^2 is zero while the function U_1^2 is given by Eq. 40. Likewise it can be seen that on the upper branch of the curve given by Eq. 25 the value of the function U_2^2 is always zero and the value of the function U_2^2 is given by Eq. 40 for positive values

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of Y.

In the same manner we obtain the values of U_1^2 and U_2^2 on the curve given by Eq. 26.

$$U_{1,2}^2 = \frac{M}{Y(Y^2-1-M^2)} \left\{ - \frac{Y^3+MY^2-Y-M/2}{Y+M} \pm \sqrt{\left(\frac{Y^3+MY^2-Y-M/2}{Y+M}\right)^2} \right\} \tag{42}$$

To determine the sign of the square root it is again necessary to study the behavior of the cubic polynomial and the roots of the equation

$$Y^3+MY^2-Y-\frac{M}{2} = 0 \tag{43}$$

It is immediately obvious that this equation becomes identical to Eq. 28 if the variable Y is replaced by -Y. The roots are then as shown on Fig. 5 only with the positive and negative Y axis interchanged. From the same considerations as before only the positive root, designated henceforth as the third root (of Eq. 43) is of interest. The third root lies between the values

$$\frac{1}{\sqrt{2}} < Y_3 < 1 \tag{44}$$

This, again, means that a point exists on the curve given by Eq. 26 with an ordiante Y_3 which satisfies the cubic of Eq. 43 and consequently both U_1^2 and U_2^2 are equal to zero in that point and, according to the argument

presented before, this point, too, lies on the curve given by Eq. 16, i.e., on the left boundary of the region of complex values of U_1^2 and U_2^2. Thus the curve of Eq. 26 has a point in common with the left boundary of the complex region given by Eq. 16, the two curves are tangent to each other at that point. We shall call this point the upper tangent point. Now for values of Y between the ordinate intercept of the curve given by the Eq. 26 and the third root, viz.,

$$-\frac{M}{2}+\sqrt{\left(\frac{M}{2}\right)^2+1} < Y < Y_3 \tag{45}$$

the squared expression in Eq. 42 is negative

$$Y^3+MY^2-Y-\frac{M}{2} < 0 \tag{46}$$

From where follows for U_1^2

$$U_1^2=\frac{-2M}{Y(Y^2-1-M^2)}\left[\frac{Y^3+MY^2-Y-M/2}{Y+M}\right] \tag{47}$$

and for U_2^2

$$U_2^2=\frac{M}{Y(Y^2-1-M^2)}\left[-\frac{Y^3+MY^2-Y-M/2}{Y+M}-\left(-\frac{Y^3+MY^2-Y-M/2}{Y+M}\right)\right]=0 \tag{48}$$

when, on the other hand, the value of Y is greater than the value of the third root the following relations hold

$$Y^3 + MY^2 - Y - \frac{M}{2} > 0 \tag{49}$$

$$Y > Y_3 \tag{50}$$

The squared quantity in Eq. 42 is then positive and we have for U_1^2

$$U_1^2 = \frac{M}{Y(Y^2-1-M^2)}\left[-\frac{Y^3+MY^2-Y-M/2}{Y+M} + \left(\frac{Y^3+MY^2-Y-M/2}{Y+M}\right)\right] = 0 \tag{51}$$

and for U_2^2

$$U_2^2 = \frac{-2M}{Y(Y^2-1-M^2)}\left[\frac{Y^3+MY^2-Y-M/2}{Y+M}\right] \tag{52}$$

In the same manner it can be shown that on the left boundary of the complex region given by Eq. 16 the values of U^2 are negative for values of Y samller than the value of the third root $(Y < Y_3)$ and positive for values of Y greater than the value of the third root $(Y_3 < Y < 1)$ and that on the right boundary given by Eq. 18 the values of U^2 are positive for values of Y smaller than the value of the second root $(Y < Y_2)$ and positive the values of Y greater than the value of the second root

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$(Y_2 < Y < 1)$. Moreover it can be seen that on the line $Y=1$ the values of functions U_1^2 and U_2^2 are as follows:

In the interval:

$$Y=1,\ 0 < X < 1 \tag{53}$$

$$U_1^2 = 0 \tag{54}$$

$$U_2^2 = 1 - X^2 \tag{55}$$

and in the interval:

$$Y=1,\ 1 < X < \infty \tag{56}$$

$$U_1^2 = 1 - X^2 \tag{57}$$

$$U_2^2 = 0 \tag{58}$$

To show that the roots of Equations 28 and 43 are also the roots of Eq. 24 obtained for the condition that both U_1^2 and U_2^2 vanish we multiply the cubic polynomials of Equations 28 and 43; the resulting bicubic polynomial is just that of Eq. 24.

Since in the complex region the functions U_1^2 and U_2^2 are complex conjugates they can be represented by their modulus ρ and argument φ, viz.; $U_1 = \rho \angle \varphi$; $U_2 = \rho \angle -\varphi$. Note that we have here U_1 and U_2 instead of U_1^2 and U_2^2. Contour curves of constant

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modulus and argument of the functions U_1 and U_2 are of greater practical value than the contour curves for constant real and imaginary part of U^2. Tables of Bessel functions of complex arguments[16] give the argument of the Bessel functions in polar form.

The modulus ρ is readily found from Eq. 13:

$$\rho^4 = \frac{1}{4(Y^2-1-M^2)^2}\left\{\left[(2(Y^2-1-M^2)+\frac{M^2}{Y^2})(X^2-Y^2)+2(Y^2-1)\right]^2 \right. \tag{59}$$

$$\left. + \frac{M^4}{Y^4}\left[4\frac{X^2Y^2}{M^2}(1-Y^2)-(X^2-Y^2)^2\right]\right\}$$

To find the equation of a contour curve in the Y-X (ω - β) plane for constant value of ρ the above equation is expanded in powers of X^2 and a quadratic equation in X^2 is obtained.

$$C_4X^4 + C_2X^2 + C_0 - \rho^4 = 0 \tag{60}$$

The coefficients C_0, C_2, C_4 depend only on Y and M and are given

by the expressions:

$$C_0 = \frac{\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right]^2 Y^4 - 4Y^2(Y^2-1)\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right] + 4(1-Y^2)^2 - M^4}{4(Y^2-1-M^2)^2} \tag{61}$$

$$C_2 = \frac{-2Y^2\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right]^2 + 4(Y^2-1)\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right] + 4\frac{M^2}{Y^2}(1-Y^2) + 2\frac{M^4}{Y^2}\Big\}}{4(Y^2-1-M^2)^2} \tag{62}$$

$$C_4 = \frac{Y^2-1-M^2+M^2/Y^2}{Y^2-1-M^2} \tag{63}$$

From Eq. 60 we obtain the explicit form of the equation for the contour curve of constant modulus ρ.

$$X = \sqrt{-\frac{C_2}{2C_4} \pm \sqrt{\left(\frac{C_2}{2C_4}\right)^2 + \frac{\rho^4 - C_0}{C_4}}} \tag{64}$$

Obviously X must be real and positive. Some representative contour curves given by the above equation are shown on Figures 6 and 8.

The argument φ is also found from Eq. 13:

$$\tan(2\varphi) = \frac{\frac{M^2}{Y^2}\sqrt{4\,\frac{X^2Y^2}{M^2}(1-Y^2) - (X^2-Y^2)^2}}{-\left[2(Y^2-1-M^2)+\frac{M^2}{Y^2}\right] - 2(Y^2-1)} \tag{65}$$

To find the equation of a contour curve for constant argument in the Y-X

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$(\omega-\beta)$ plane we use the notation $T = \tan(2\varphi)$ and expand the Eq. 65 in powers of X^2 so that a quadratic equation in X^2 is obtained.

$$C_4' X^4 + C_2' X^2 + C_0' = 0 \qquad (66)$$

The coefficients C_0', C_2', C_4' depend only on Y, M, and T and are given by the following expressions:

$$C_0' = T^2 Y^4 \left[2(Y^2-1-M^2) + \frac{M^2}{Y^2} \right]^2 - 4T^2(Y^2-1)Y^2 \left[2(Y^2-1-M^2) + \frac{M^2}{Y^2} \right] + 4T^2(Y^2-1)^2 + M^4 \qquad (67)$$

$$C_2' = -2Y^2T^2 \left[2(Y^2-1-M^2) + \frac{M^2}{Y^2} \right]^2 + 4T^2(Y^2-1) \left[2(Y^2-1-M^2) + \frac{M^2}{Y^2} \right] - \frac{4M^2}{Y^2}(1-Y^2) \qquad (68)$$

$$C_4' = T^2 \left[2(Y^2-1-M^2) + \frac{M^2}{Y^2} \right]^2 + \frac{M^4}{Y^4} \qquad (69)$$

From Eq. 66 we obtain the explicit form of the equation for the contour curve for constant argument.

$$X = \sqrt{ -\frac{C_2'}{2C_4'} \pm \sqrt{ \left(\frac{C_2'}{2C_4'} \right)^2 - \frac{C_0'}{C_4'} } } \qquad (70)$$

Again, X must be real and positive. Some representative contour curves given by the above equation are shown on Figures 7 and 9.

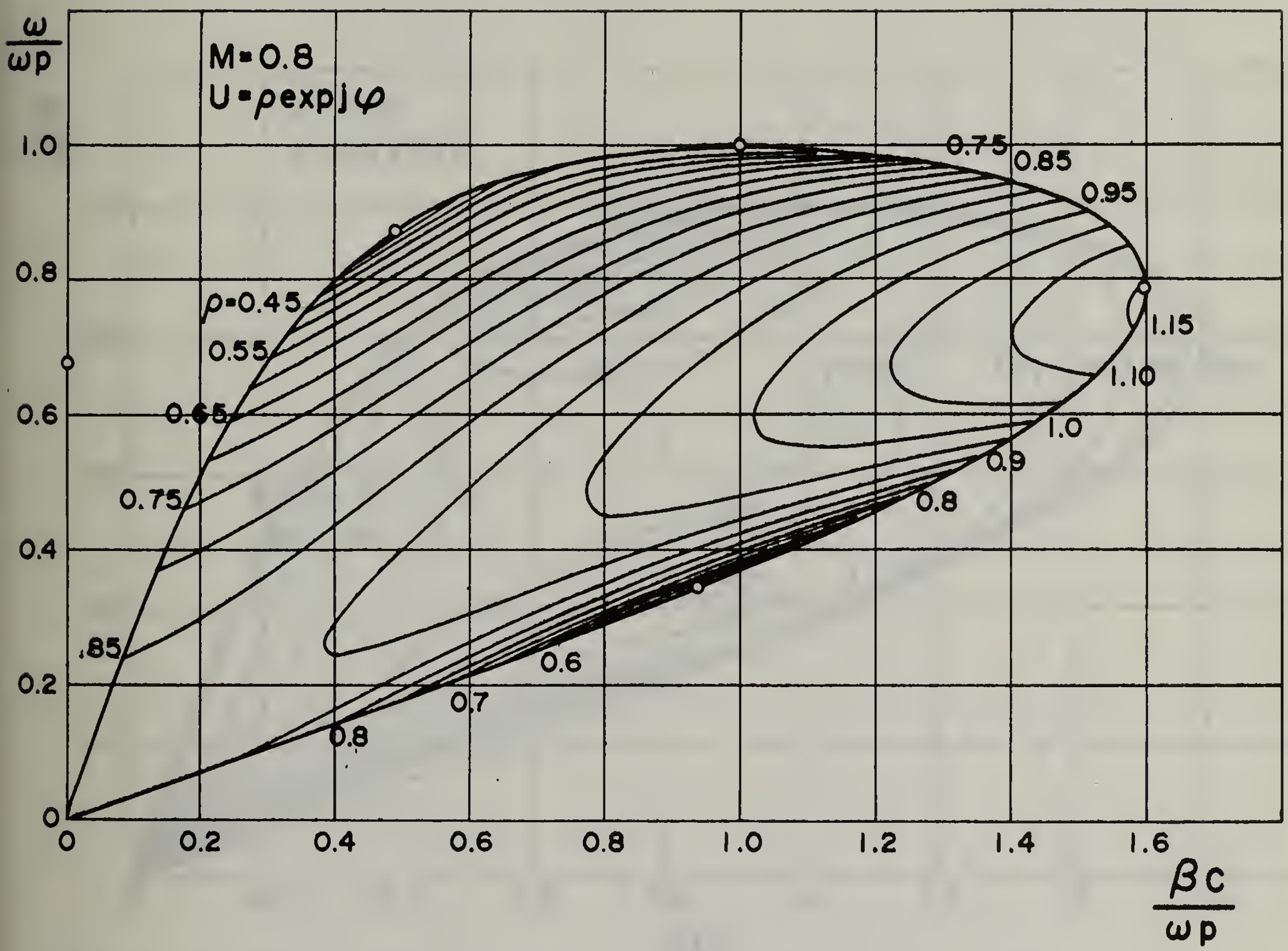
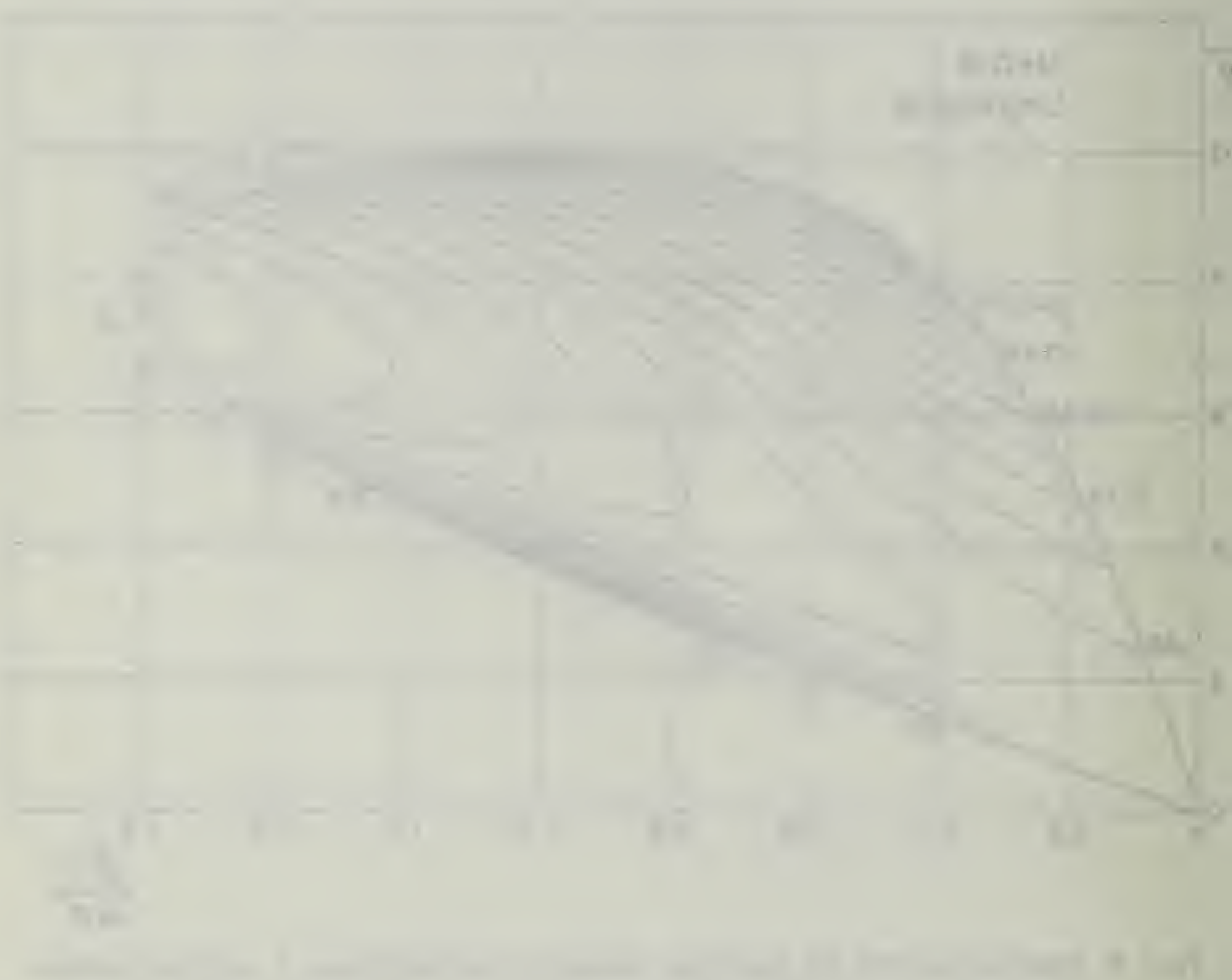


Fig. 6 Contour curves for constant values of the modulus ρ of the function $U = \rho \angle \varphi$ in the complex region of the $\omega - \beta$ plane at the value of $M = 0.8$.



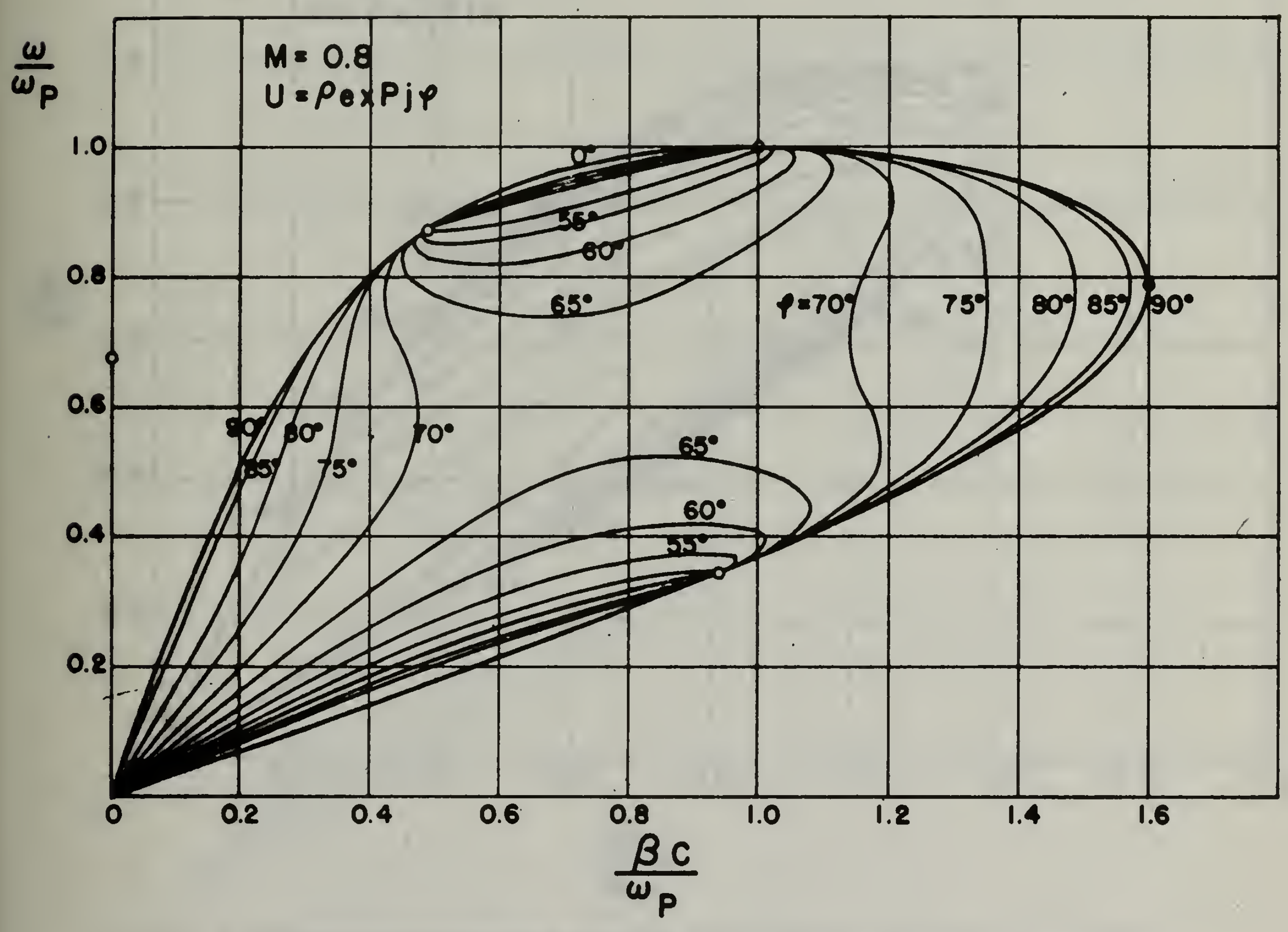


Fig. 7 Contour curves for constant values of the argument φ of the function $U = \rho \angle \varphi$ in the complex region of the $\omega - \beta$ plane at the value of $M = 0.8$.

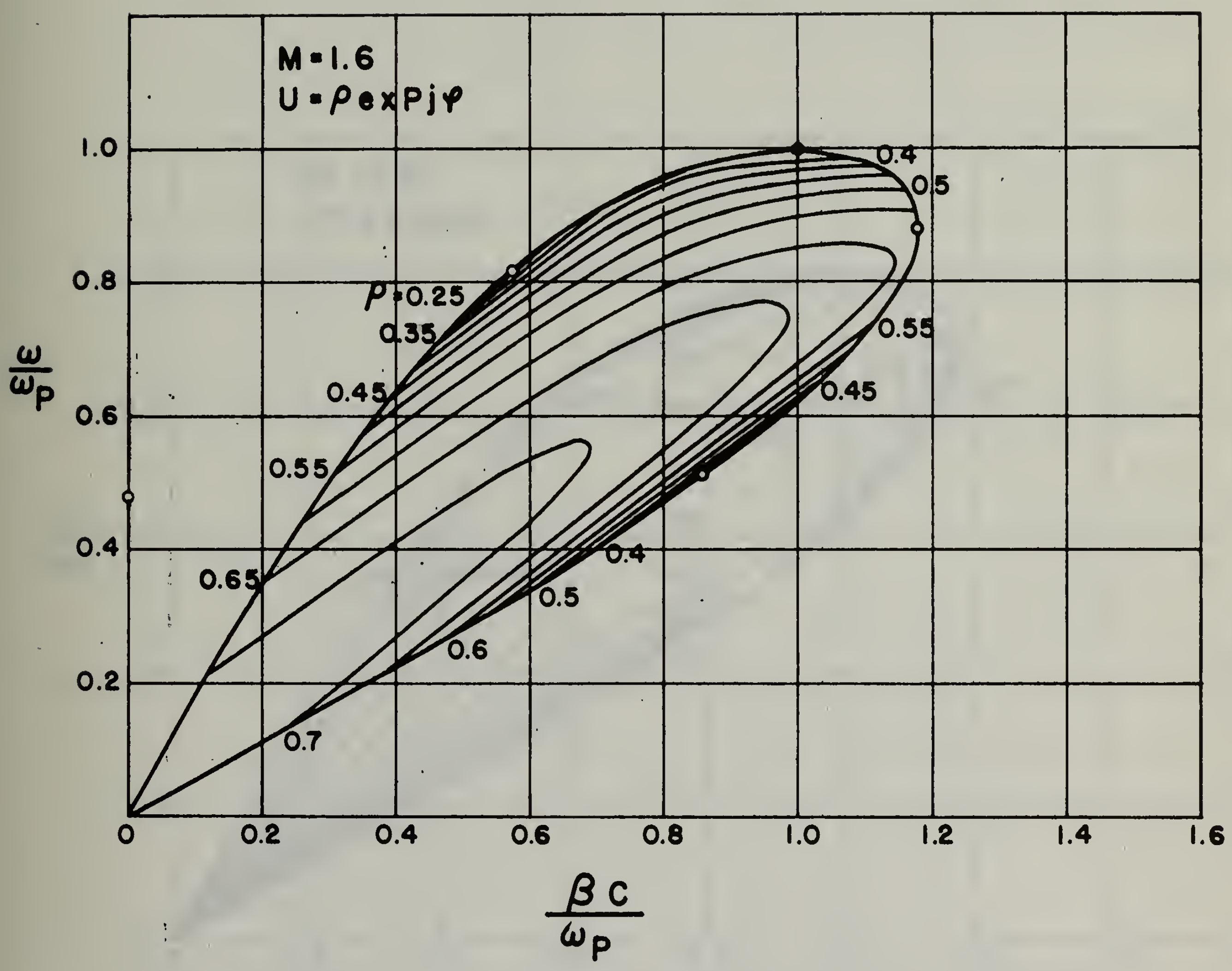


Fig. 8 Contour curves for constant values of the modulus ρ of the function U = $\angle\varphi$ in the complex region of the ω − β plane at the value of M = 1.6.

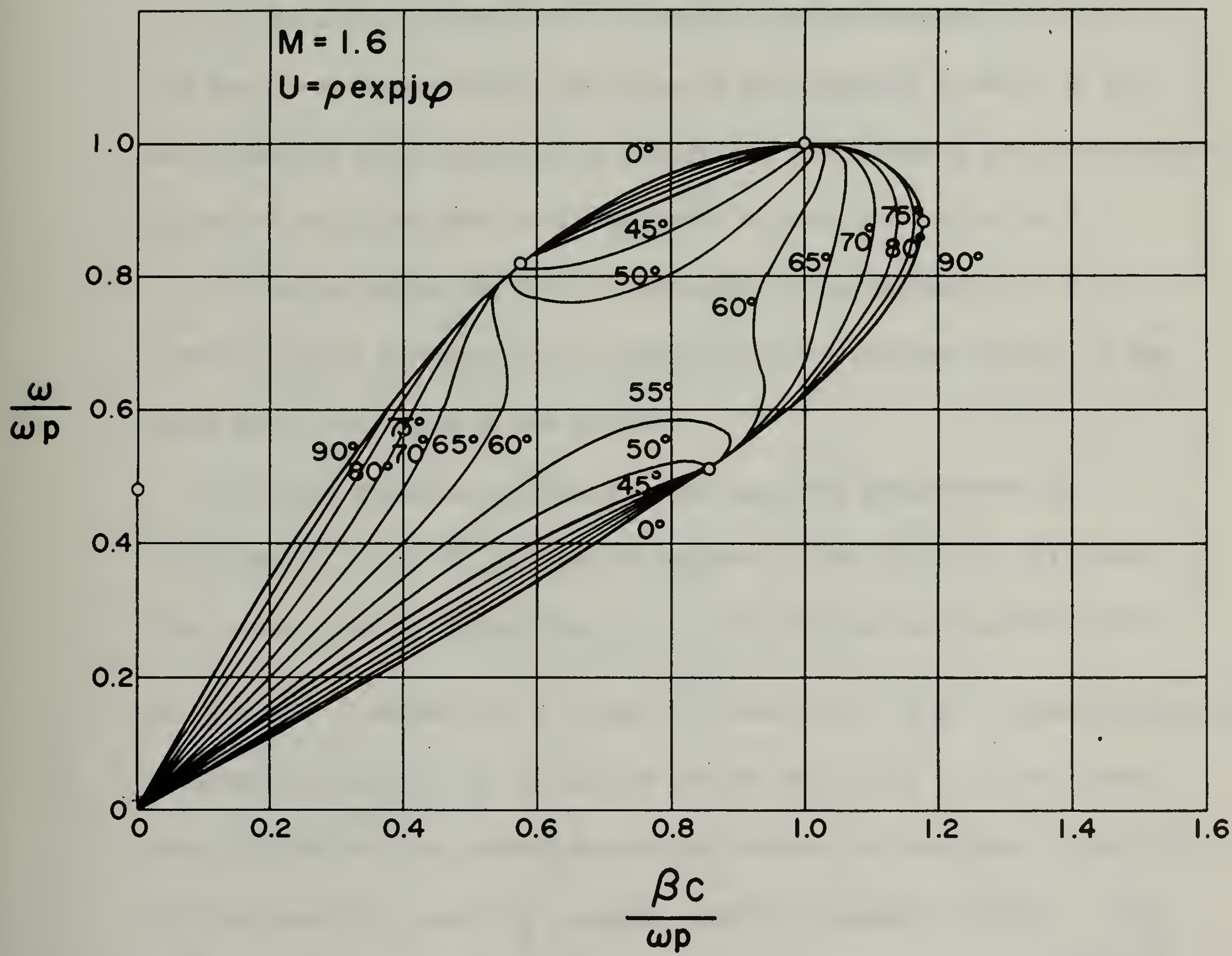


Fig. 9 Contour curves for constant values of the argument φ of the function $U = \rho\angle\varphi$ in the complex region of the $\omega - \beta$ plane at the value of $M = 1.6$.

It is interesting to note the curves delineating various zones in the complex region. On the left boundary of the complex region between the origin and the upper tangent point the value of the argument of U is $\pi/2$, in the upper tangent point the value of the argument is not defined and can assume any value, and between the upper tangent point and the point (X=1, Y=1) the value of the argument is zero. In this point, again, the argument is not defined.

On the right boundary of the complex region between the origin and the lower tangent point the value of the argument is zero, in the lower tangent point its value is not defined and between the lower tangent point and the point (X=1, Y=1) the value of the argument is $\pi/2$.

On curves where $\mathrm{Re}(U^2) = 0$ the value of the argument is $\pi/4$. Contour curves converge in the upper and lower tangent points, in the point (X=1, Y=1), and in the origin.

It is now possible to obtain a picture of the behavior of the functions U_1^2 and U_2^2 in various regions of the Y-X $(\omega - \beta)$ plane. The curves given by Equations 14, 15. 25, 26 and the lines Y=1 and $Y = \sqrt{1+M^2}$ divide the Y-X $(\omega - \beta)$ plane into 12 or 13 zones depending whether the value of M is less or greater than unity. It has already been pointed out that within the region defined by Equations 14 and 15 the functions U_1^2 and U_2^2 assume complex conjugate values. In the other regions the following has been found concerning the sign of U_1^2 and U_2^2:

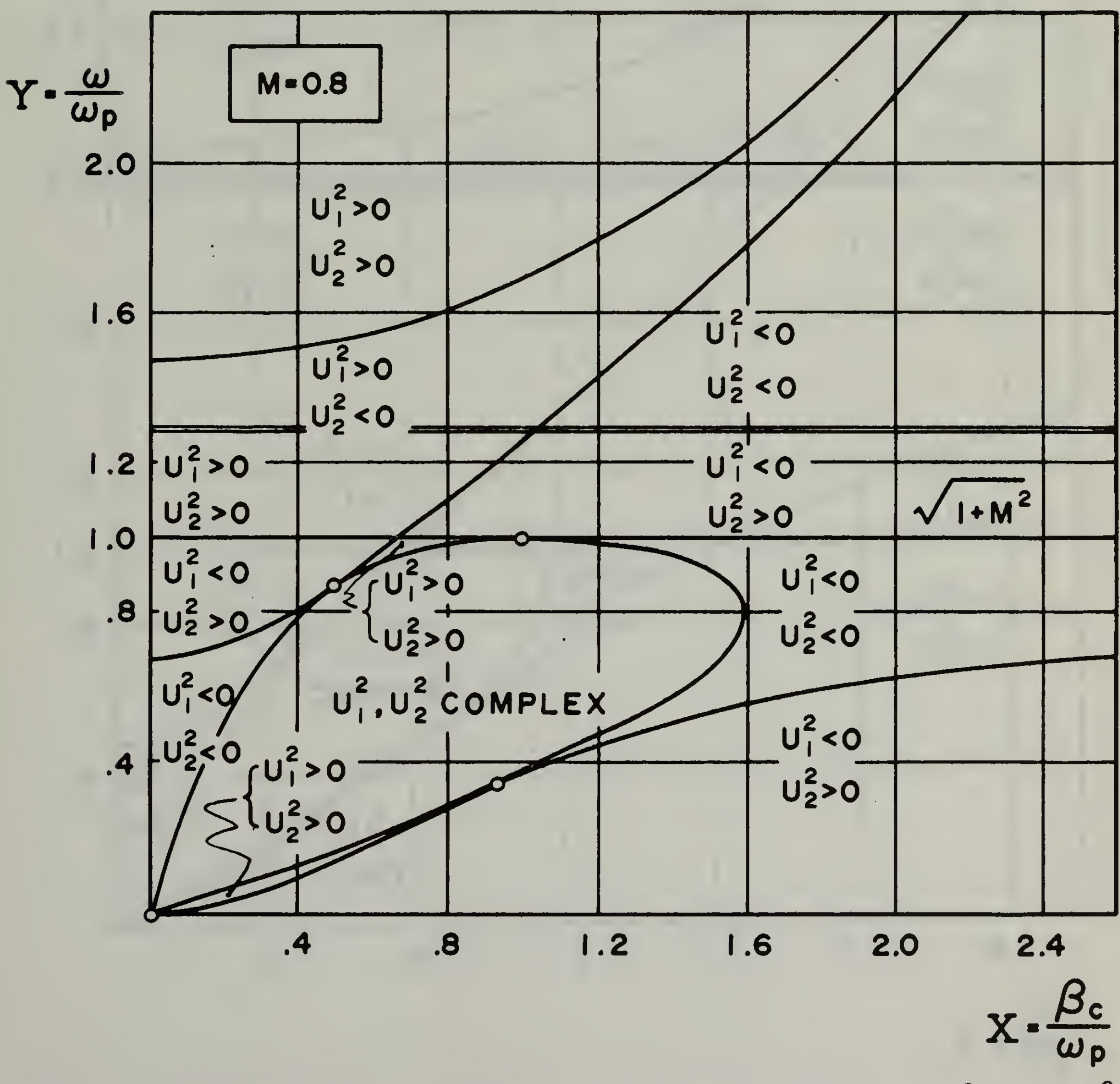
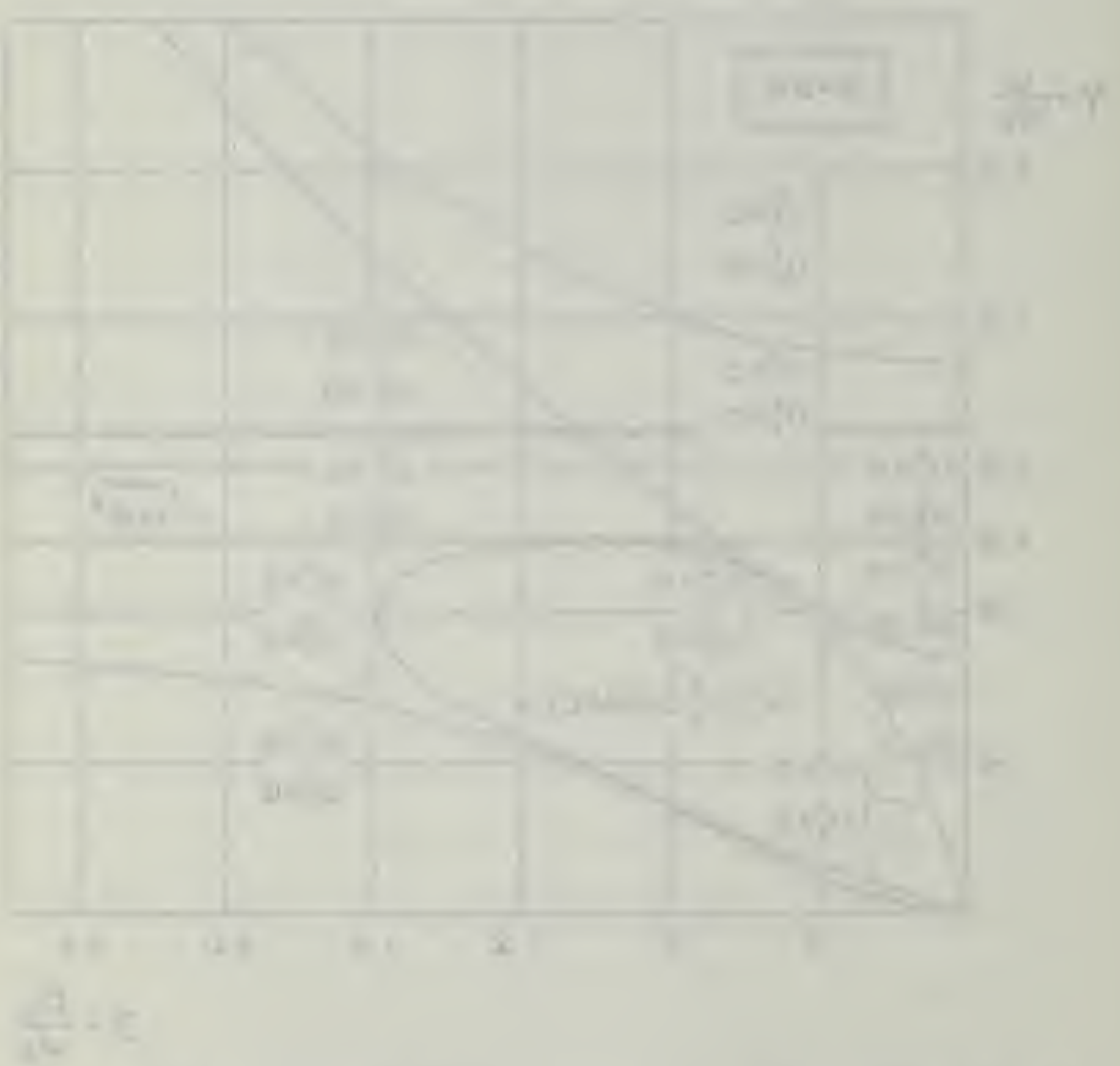


Fig. 10 Zones in the $\omega - \beta$ plane in which the functions U_1^2 and U_2^2 are of the same or of opposite sign. General behavior for values of $M < 1$.



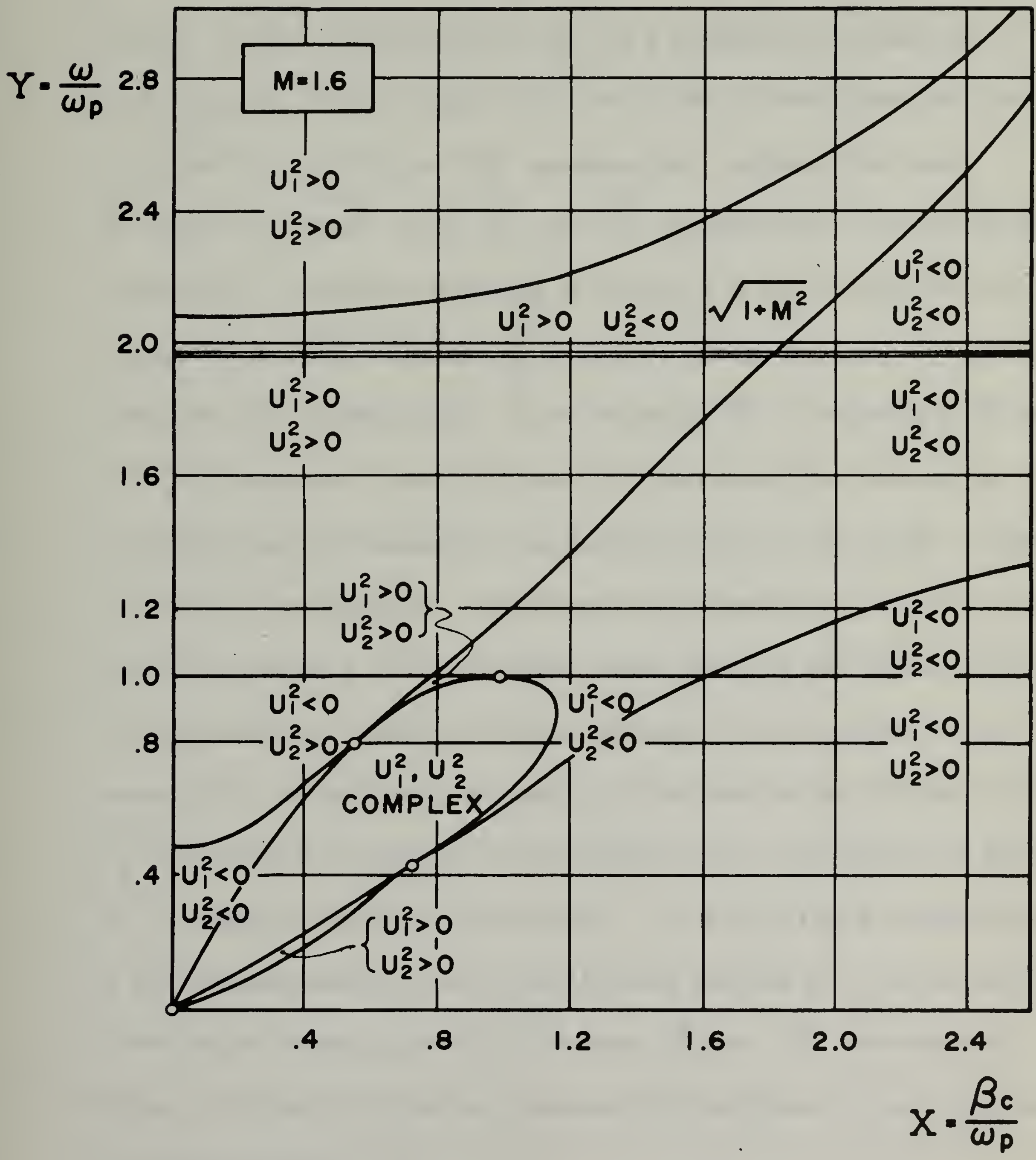
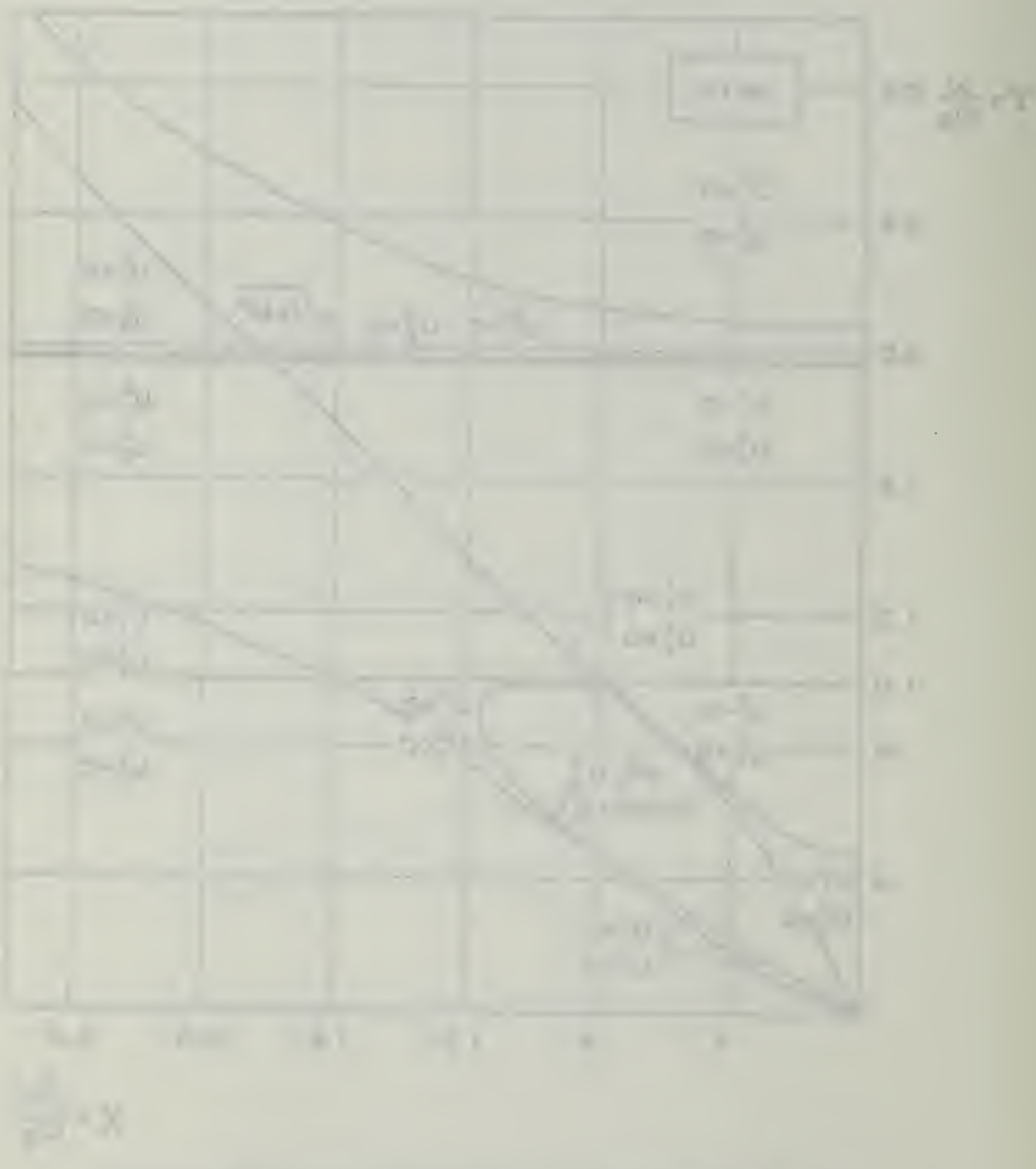


Fig. 11 Zones in the $\omega - \beta$ plane in which the functions U_1^2 and U_2^2 are of the same or of opposite sign. General behavior for values of $M > 1$.



At Values of M smaller than unity, in the region above the upper branch of the curve given by Eq. 25. Both U_1^2 and U_2^2 are positive. Between the upper branch of the curve given by Eq. 25, and the curve given by Eq. 26, above the straight line $Y=\sqrt{1+M^2}$; $U_1^2 > 0$, $U_2^2 < 0$. On the line $Y=\sqrt{1+M^2}$ the function U_2^2 has a pole while the function U_1^2 is continuous. To the right of the curve of Eq. 26 and above the line $Y=\sqrt{1+M^2}$ both U_1^2 and U_2^2 are negative. Between the lines $Y=1$ and $Y=\sqrt{1+M^2}$ both U_1^2 and U_2^2 are positive to the left of the curve of Eq. 26 while to the right of the curve of Eq. 26 the value of U_1^2 is negative and the value of U_2^2 positive. Under the line $Y=1$ and above the curve given by Eq. 26 the value of U_1^2 is negative while that of U_2^2 is positive. Under the line $Y=1$ and under the curve of Eq. 26 and above the left boundary of the complex region given by Eq. 16 both U_1^2 and U_2^2 are positive. Under the curve given by Eq. 26 and to the left of the boundary of this complex region given by Eq. 16 both U_1^2 and U_2^2 are negative. Between the right boundary of the complex region given by Eq. 18 and the lower branch of the curve of Eq. 25 both U_1^2 and U_2^2 are positive. Under the lower branch of the curve of Eq. 25 the function U_1^2 is negative while U_2^2 is positive. To the right of the right boundary of the complex region given by Eq.18 under the line $Y=1$ and above the lower branch of the curve of Eq. 25 both U_1^2 and U_2^2 are negative. Figure 10 shows the zones and signs of the functions U_1^2 and U_2^2 for the case when $M < 1$.

For values of M greater than unity the lower branch of the curve given by Eq. 25 and the line $Y=1$ intersect and modify the picutre with the addition of another zone. In such cases the behavior of functions U_1^2 and U_2^2 is modified as follows. To the right of the lower branch of the curve of Eq. 25 and below the line $Y=1$ the function U_1^2 is negative while U_2^2 is positive. In the newly formed triangular zone bounded by the right boundary of the complex region given by Eq. 18, the line $Y=1$, and the lower branch of the curve given by Eq. 25 both U_1^2 and U_2^2 are negative, likewise they are negative in the region above the line $Y=1$ and below the lower branch of the curve given by Eq. 25. In the region to the right of the curve of Eq. 26, above the line $Y=1$ and above the lower branch of the curve of Eq. 25 and below the line $Y=\sqrt{M^2+1}$ the function U_1^2 is negative while the function U_2^2 is positive. The behavior of U_1^2 and U_2^2 in the other zones remains the same as for cases when $M < 1$. Figure 11 shows the zones and signs of functions U_1^2 and U_2^2 for this case $(M > 1)$.

The partition of the $\omega - \beta$ (Y-X) plane into zones as described above is quite general and is valid for all possible cases. These cases fall under one of the two types, viz., those with $M < 1$ and those with $M > 1$. There is, however, some variation in the sequence of special frequencies $(Y_2, Y_3, -\frac{M}{2} + \sqrt{\frac{M^2}{4}+1}, \sqrt{M^2+1}$, etc.) depending in which interval lies the value of M. The sequential order of these

special frequencies depend only on the values of M. The possible cases are listed below.

$$0 < M < \frac{1}{\sqrt{2}} \tag{71}$$

$$0 < Y_2 < M < -\frac{M}{2} + \sqrt{\frac{M^2}{4}+1} < Y_3 < 1 < \sqrt{M^2+1} < \frac{M}{2} + \sqrt{\frac{M^2}{4}+1} \tag{72}$$

$$\frac{1}{\sqrt{2}} < M < 1 \tag{73}$$

$$0 < Y_2 < -\frac{M}{2} + \sqrt{\frac{M^2}{4}+1} < Y_3 < M < 1 < \sqrt{M^2+1} < \frac{M}{2} + \sqrt{\frac{M^2}{4}+1} \tag{74}$$

$$1 < M < 1.5 \tag{75}$$

$$0 < Y_2 < -\frac{M}{2} + \sqrt{\frac{M^2}{4}+1} < Y_3 < 1 < M < \sqrt{M^2+1} < \frac{M}{2} + \sqrt{\frac{M^2}{4}+1} \tag{76}$$

$$1.5 < M \tag{77}$$

$$0 < -\frac{M}{2} + \sqrt{\frac{M^2}{4}+1} < Y_2 < Y_3 < 1 < M < \sqrt{M^2+1} < \frac{M}{2} + \sqrt{\frac{M^2}{4}+1} \tag{78}$$

In addition to the general behavior of the functions U_1^2 and U_2^2 or, if roots are taken, U_1 and U_2, it is also desirable to have contour representations of these functions in the $\omega - \beta$ (Y-X) plane where U_1

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and U_2 have either real or imaginary values, i.e., outside of the complex region.

The equation of contour curves in the $\omega - \beta$ (Y-X) plane for which U is a real or an imaginary constant can be readily obtained by solving Eq. 3 for X.

$$X^2 = Y^2 - U^2 + \frac{M^2U^2 + 2Y^2(1-Y^2)}{2(Y^2-1)(Y^2-M^2)} \pm \frac{M\sqrt{4Y^2(Y^2-1)(Y^2-1-U^2)+M^2U^4}}{2(Y^2-1)(Y^2-M^2)} \tag{79}$$

This is a simple solution of a biquadratic equation in X. For contour curves in the first quadrant of the $\omega - \beta$ (Y-X) plane only positive values of X^2 are acceptable. There appears to be no simple rule to determine whether the curves obtained from Eq. 79 are the contour curves of U_1 or U_2. This question can be resolved by calculating with the obtained values of Y and X the desired function U_1 or U_2, as the case may be, using the rule concerning the selection of the sign as stated above and then determining whether the calculated value is equal to the value of U originally set in Eq. 79. If the calculated value of U_1 or U_2 is equal to the value of U set in Eq.79 the point in question lies on the curve U_1 = constant or U_2 = constant respectively.

Equation 3 can also be solved with respect to Y albeit not so readily since the arrangement in powers of Y leads to a biquartic equation. This biquartic equation can have complex roots and the selection of proper curves is even more laborious in this case. There appears to be no point

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in solving the Eq. 3 for Y, therefore, since the solution with respect to X can be obtained by simple algebra.

Some representative contour curves for various real and imaginary values of U_1 and U_2 are shown on Figures 12, 13, 14, and 15.

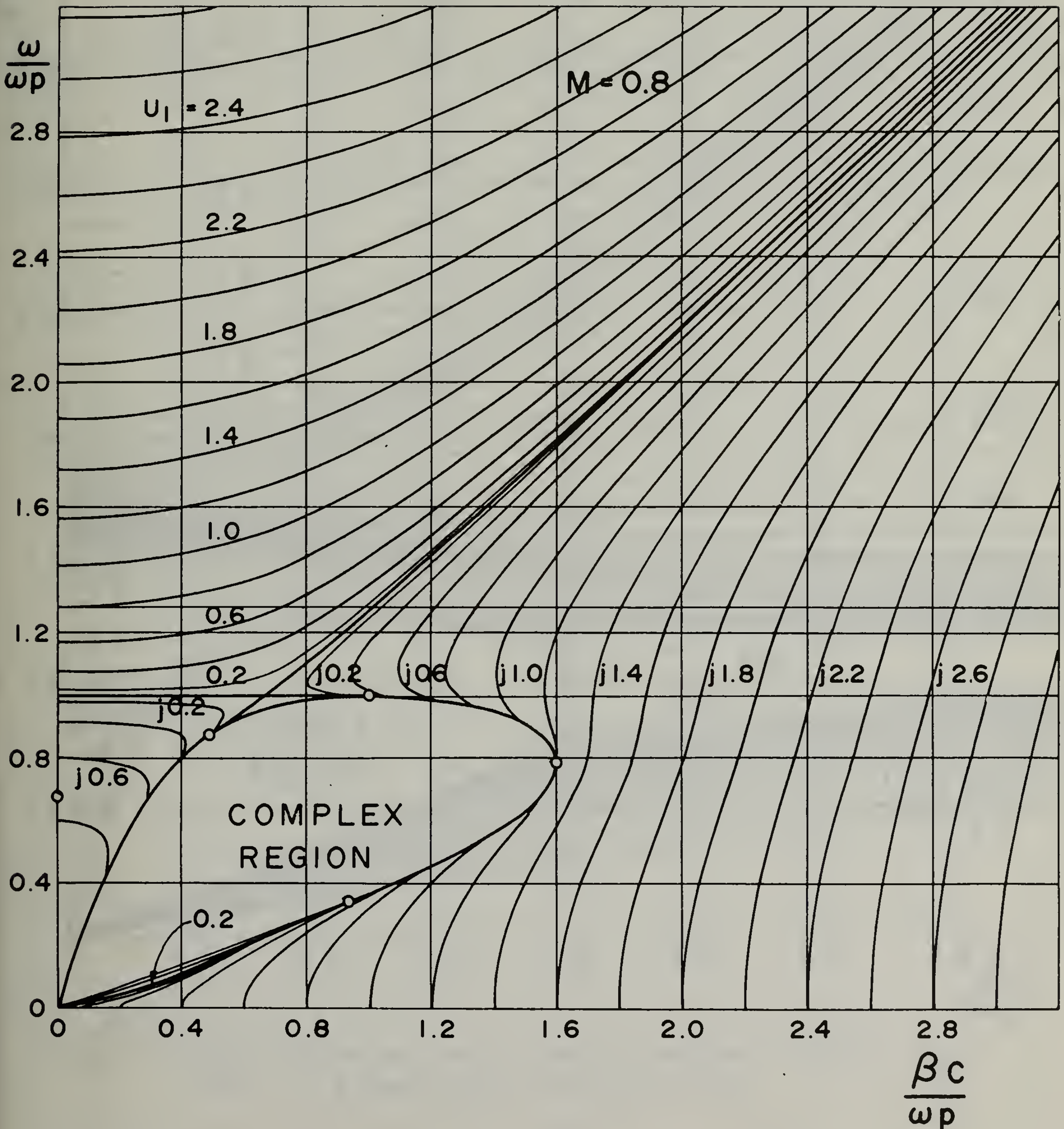
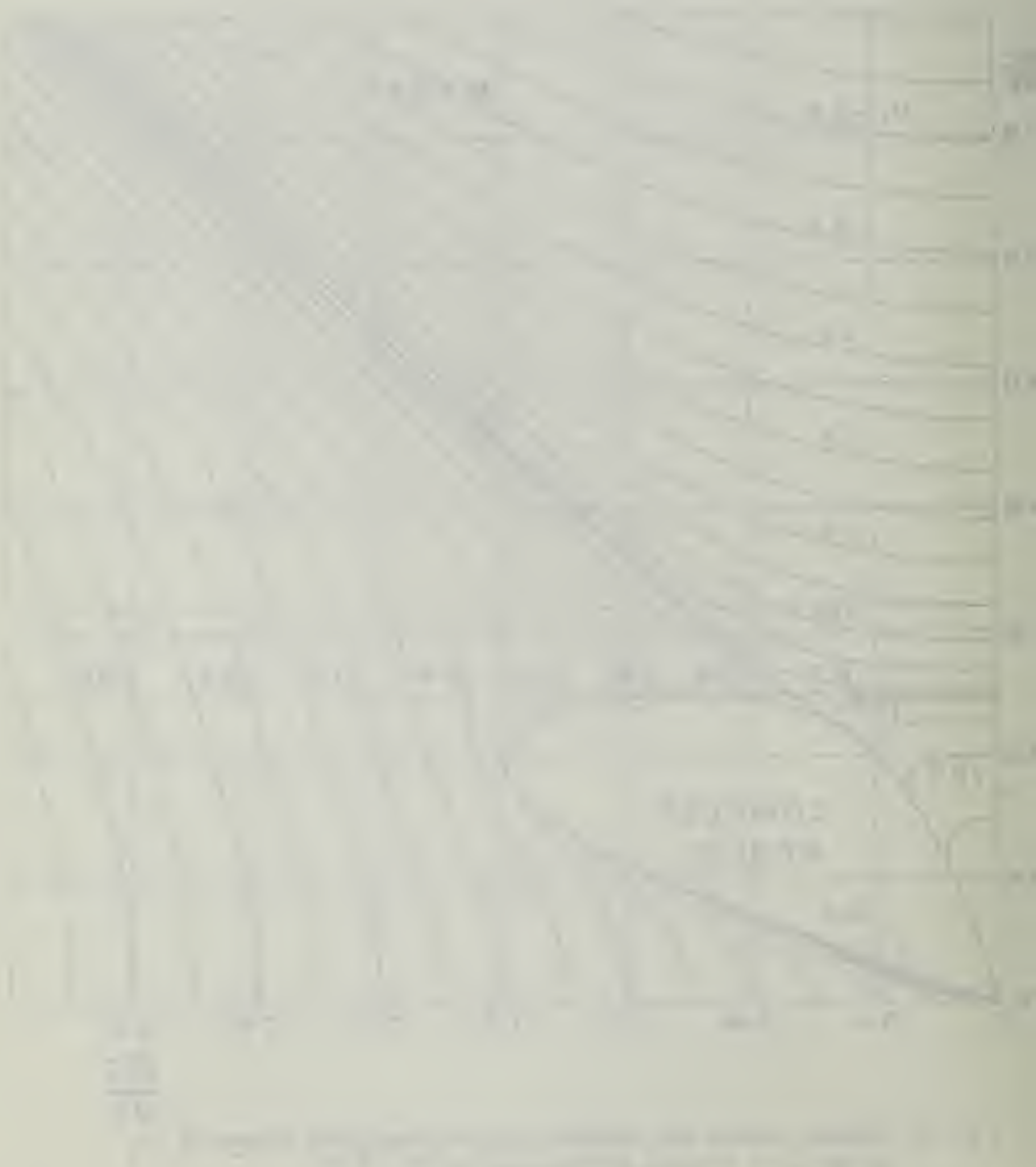


Fig. 12 Contour curves for constant real and imaginary values of U_1 in the $\omega - \beta$ plane at the value of $M = 0.8$.



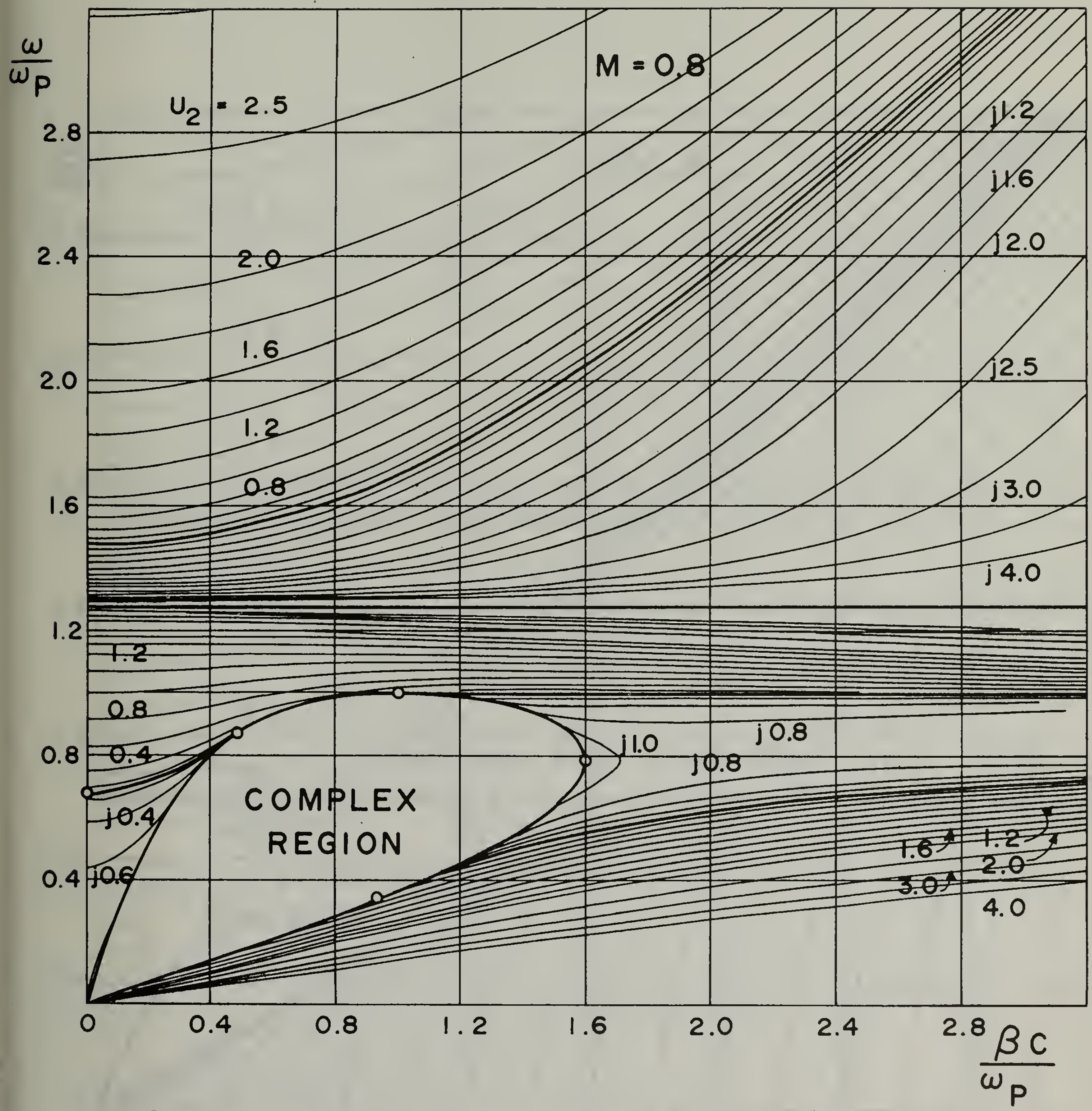


Fig. 13 Contour curves for constant real and imaginary values of U_2 in the $\omega - \beta$ plane at the value of $M = 0.8$.

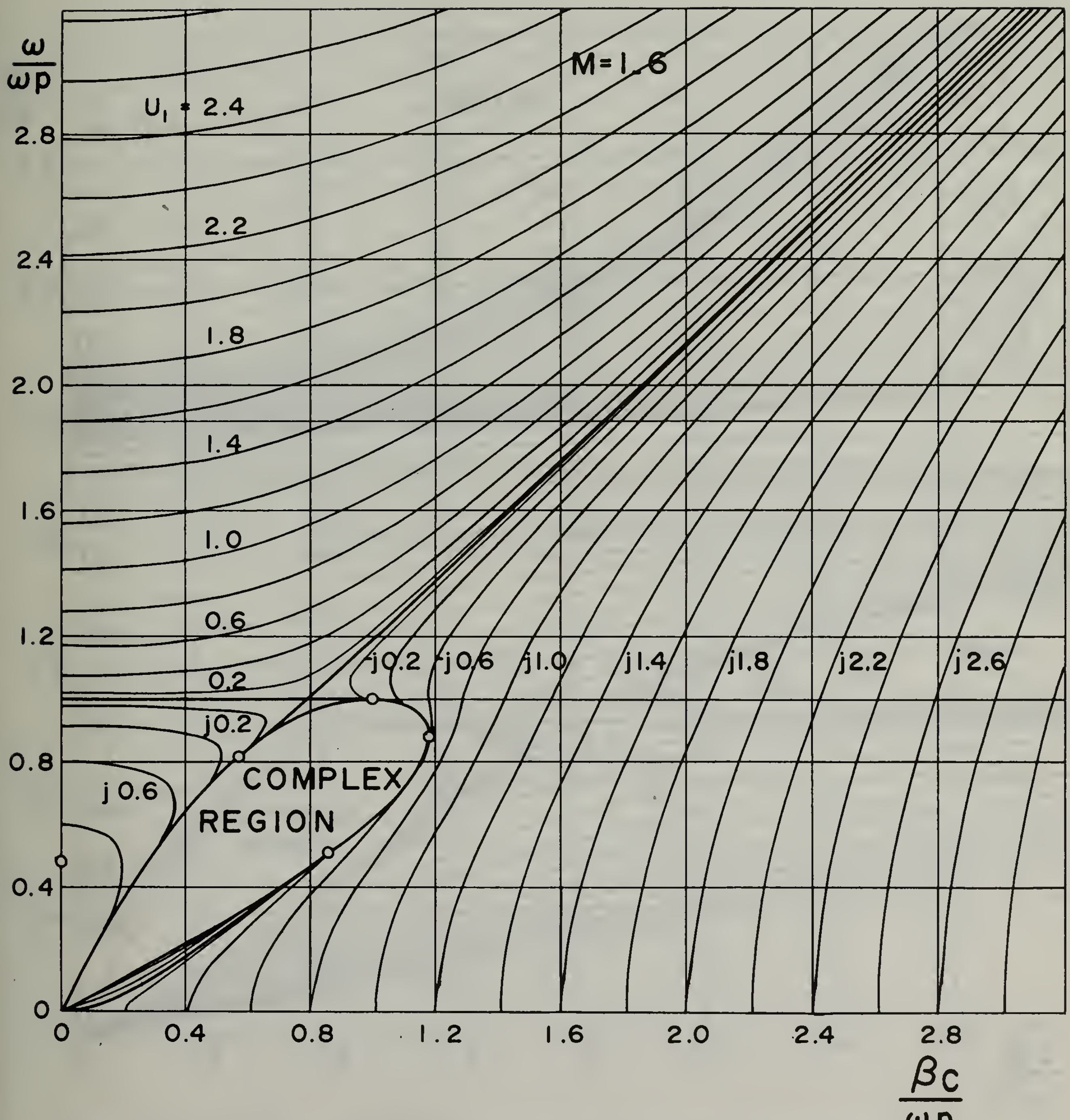
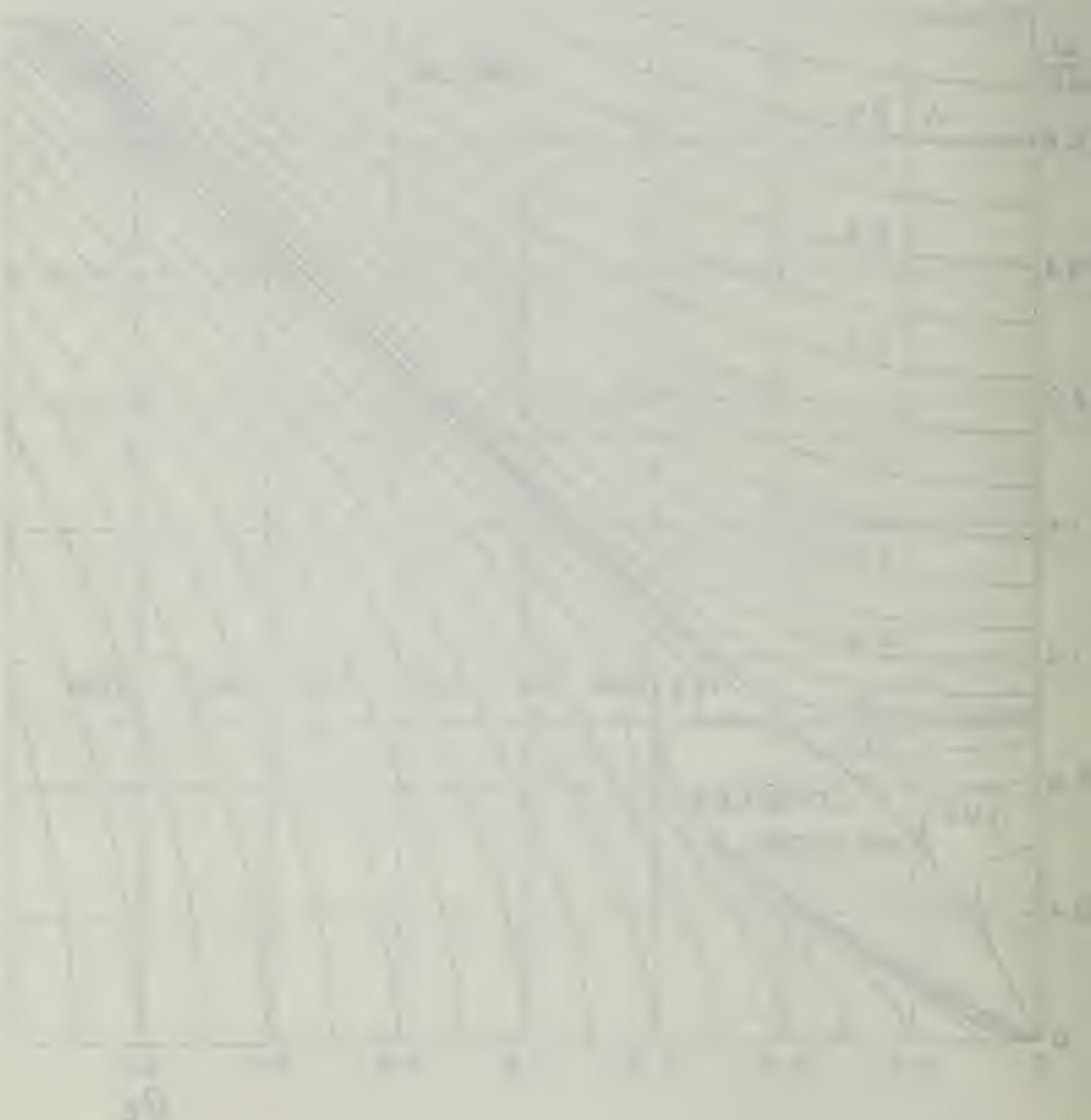


Fig. 14 Contour curves for constant real and imaginary values of U_1 in the $\omega - \beta$ plane at the value of $M = 1.6$.



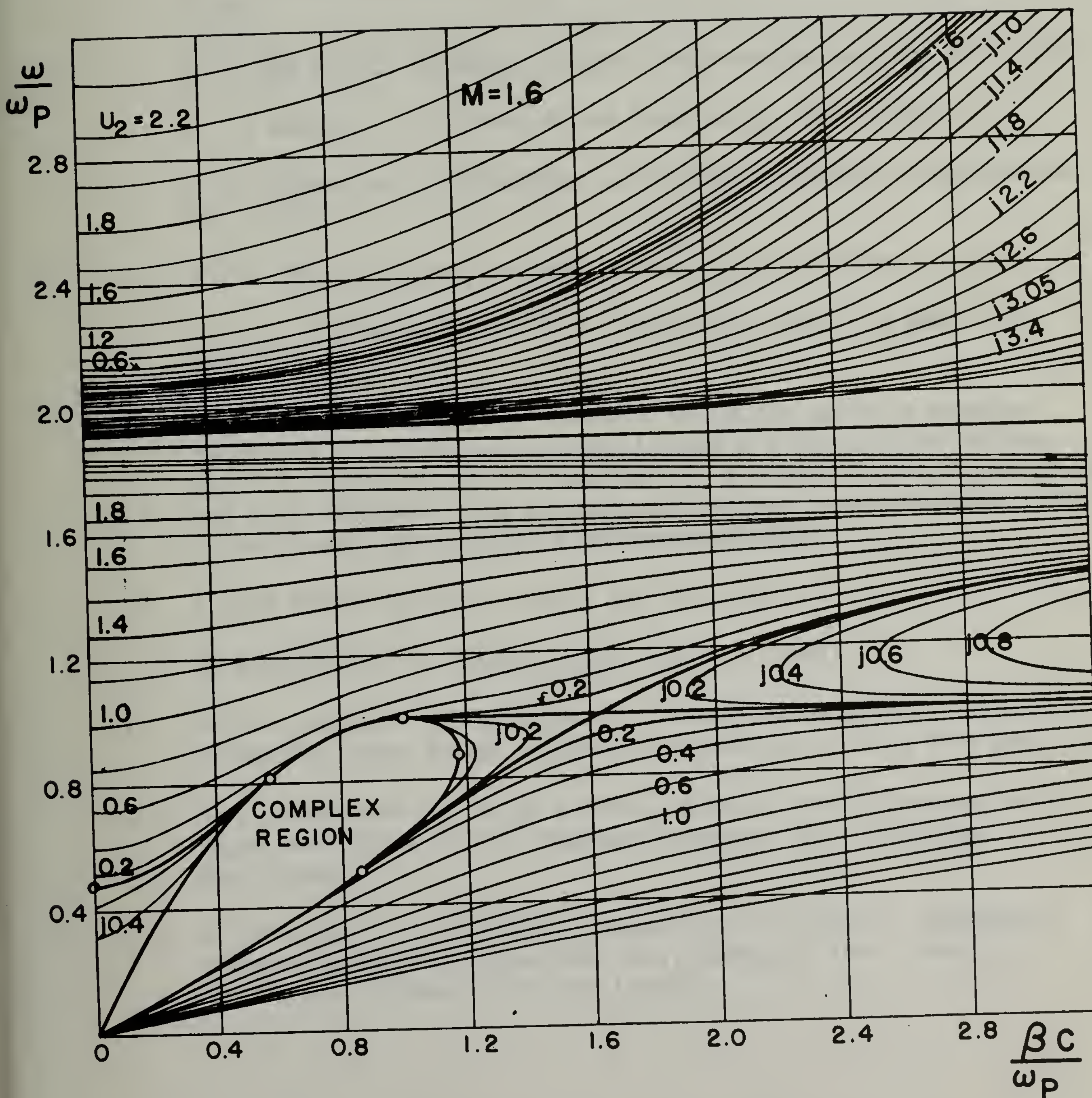


Fig. 15 Contour curves for constant real and imaginary values of U_2 in the $\omega - \beta$ plane at the value of $M = 1.6$.

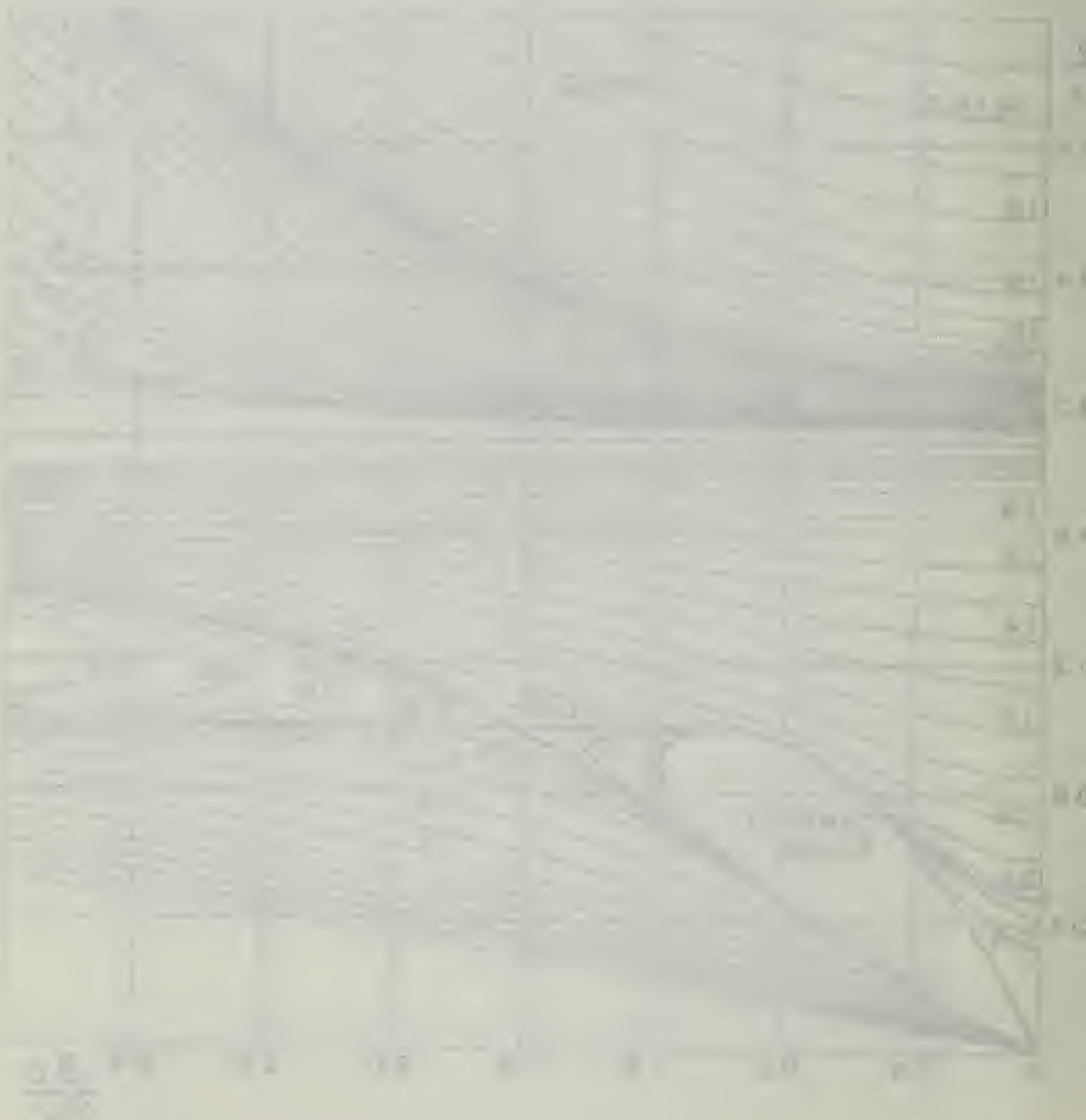


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Fig. 13 Contour curves for constant real and imaginary values of U_2 in the $\omega - \beta$ plane at the value of $M = 0.8$.

Fig. 14 Contour curves for constant real and imaginary values of U_1 in the $\omega - \beta$ plane at the value of $M = 1.6$.

Fig. 15 Contour curves for constant real and imaginary values of U_2 in the $\omega - \beta$ plane at the value of $M = 1.6$.

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1. ORIGINATING ACTIVITY (Corporate author) U. S. Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified	
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3. REPORT TITLE Behavior of Gyrotropic Plasma Separation Constants as Functions in the $\omega-\beta$ Plane		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5. AUTHOR(S) (Last name, first name, initial) Bevc, Vladislav		
6. REPORT DATE February 1965	7a. TOTAL NO. OF PAGES 54	7b. NO. OF REFS 16
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) Research Paper No. 47	
b. PROJECT NO.		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		
10. AVAILABILITY/LIMITATION NOTICES Qualified requesters may obtain copies of this report from DDC.		
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The type of the functions appearing in the expression for electromagnetic fields in finite magnetoplasmas depends only on the properties of the medium manifested in the separation constants. The $\omega-\beta$ plane of the Brillouin diagram is partitioned into a definite number of zones characterized by the type of field solutions. The general quantitative behavior of the separation constant for finite magnetoplasma in a longitudinal magnetic field on the Brillouin diagram is described.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Electronic Waveguides Wave Propagation in Electronic Waveguides Bounded Gyrotropic Plasmas Birefringent Media						

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